Turbulent Stirring and Mixing

Neither "stirring" nor "mixing" appears in the 1961 Proceedings.

Only L.S.G. Kovasznay is on record as having mentioned the word 'scalar': "Measurements of scalar fluctuations, i.e., temperature, would present the simplest case (of dispersion)".

There was a closely related session on "Diffusion and Lagrangian effects"

President: S. Corrsin
Secretaries: J.L. Lumley and P.G. Saffman
Speakers: J.L. Lumley, S. Corrsin, P.G. Saffman and J.O. Hinze

Titles of talks

- J.L. Lumley: The mathematical nature of the problem of relating Lagrangian and Eulerian statistical functions in turbulence
- S. Corrsin: Theories of turbulent dispersion
- P.G. Saffman: Some aspects of the effects of the molecular diffusivity in turbulent dispersion
- J.O. Hinze: Dispersion in turbulent shear flow

Themes

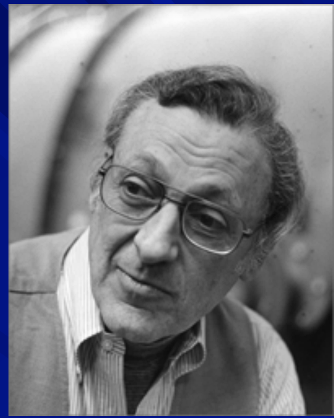
- Single particle diffusion: Long-time and medium-time behaviors
- Two-particle dispersion: Applying the Kolmogorov phenomenology, deriving Richardson's law, etc
- Shear dispersion

Yeung & Sawford

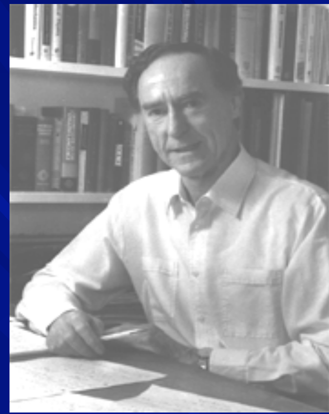
Heavily based on G.I. Taylor (1921, 1954)

Obukhov

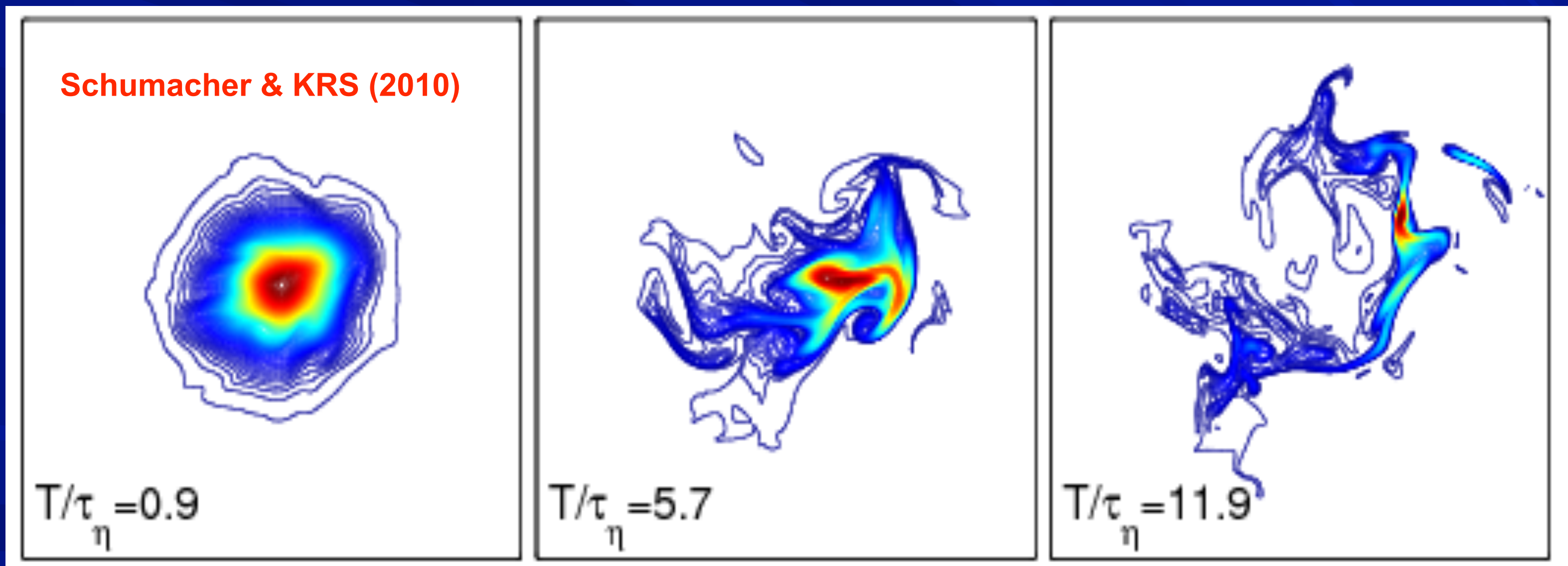
Yaglom



Corrsin



Batchelor



Prasad & KRS, *Phys. Fluids A* **2**, 792 (1990)
P. Constantin, I. Procaccia & KRS, *Phys. Rev. Lett.* **67**,1739 (1991)

Additives as passive scalars

If the velocity of advection **u**(**x**;t) solves NS = 0 without any dependence on the additive, the additive is called Passive Scalar, which obeys the

Advection diffusion equation

$$\partial\theta/\partial t + \mathbf{u}.\nabla\theta = \kappa\nabla^2\theta$$

θ(**x**;t), the additive; κ, its diffusivity (usually small); **u**(**x**;t), the advection velocity; no source terms here

The equation is linear with respect to θ.

BCs (perhaps mixed) are almost always linear as well.

Linearity holds for each realization but the equation is statistically nonlinear because of $\langle \mathbf{u}.\nabla\theta \rangle$, etc.

Bos et al.

Langevin equation

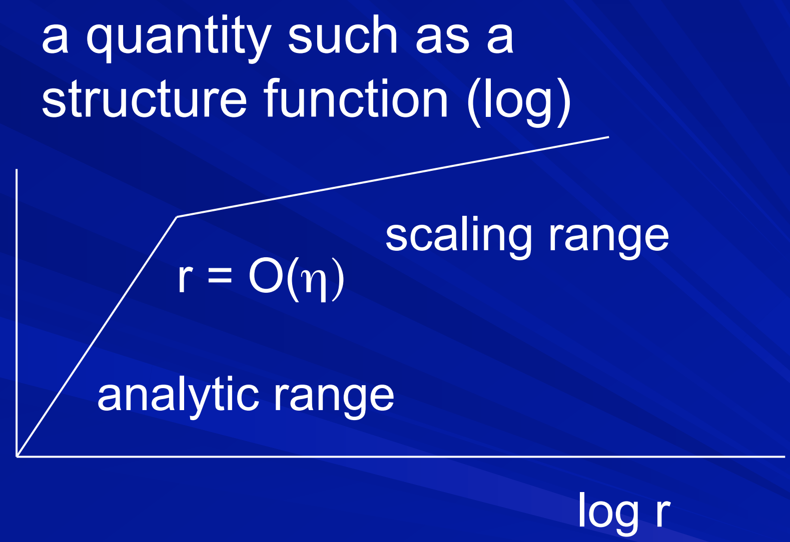
$$d\mathbf{X} = \mathbf{u}[\mathbf{X}(t);t]\, dt + (2\kappa)^{1/2}\, d\chi(t), \quad \mathbf{X}(t=0) = \mathbf{x}_0$$

$\chi(t)$ = vectorial Brownian motion, statistically independent in three components

For smooth velocity fields, single-particle diffusion as well as two-particle dispersion are well understood.

The turbulent velocity field is analytic only in the range $r < \eta$, and Hölder continuous, or "rough," in the scaling range ($\Delta_r u \sim r^h$, $h < 1$).

$h = 1/3$ for Kolmogorov turbulence $<\Delta_r u^3> \sim r$ but has a distribution in practice. "multiscaling"



C. Meneveau & KRS, *J. Fluid Mech*. **224**, 429 (1991); KRS, *Annu. Rev. Fluid Mech*. **23**, 539 (1991)

If $\Delta_r u \sim r^h$ for $h < 1$, we get $r(t) \sim t^{1/(1-h)}$, and Lagrangian paths separate explosively and are not unique; this introduces various complexities.

Model studies

- Assume some artificial velocity field satisfying div **u** = 0
- see A.J. Majda & P.R. Kramer, *Phys. Rep.* **314**, 239 (1999)

Broad-brush summary of "large-scale, long-time" results

1. For smooth velocity fields (e.g., periodic and deterministic), homogenization is possible. That is,

   $$\langle \mathbf{u}(\mathbf{x};t)\,\nabla\theta\rangle = (\kappa_T \cdot \nabla(\theta(\mathbf{x};t))$$

   where κ_T is an effective diffusivity (Varadhan, Papanicolaou, Majda, and others)
2. Velocity is a homogeneous random field, but a scale separation exists: $L_u/L_\theta \ll 1$. Homogenization is possible here as well.
3. Velocity is a homogeneous random field but delta correlated in time, $L_u/L_\theta = O(1)$; eddy diffusivity can be computed.
4. For the special case of shearing velocity (with and without transverse drift), the problem can be solved essentially completely: eddy diffusivity, anomalous diffusion, etc., can be calculated without any scale separation.

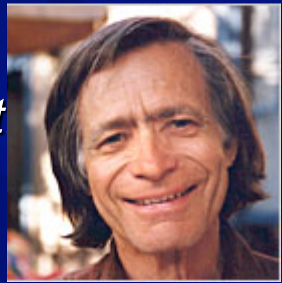
See, e.g., G. Glimm, B. Lundquist, F. Pereira, R. Peierls, *Math. Appl. Comp.* **11**, 187 (1992); M. Avellaneda & A.J. Majda, *Phil. Trans. Roy. Soc. Lond. A* **346**, 205 (1994); G. Ben Arous & H. Owhadi, *Comp. Math. Phys.* **237**, 281 (2002)

Kraichnan model

(with focus on small-scales)

R.H. Kraichnan, *Phys. Fluids* **11**, 945 (1968); *Phys. Rev. Lett* **72,** 1016 (1994)

Review: G. Falkovich, K. Gawedzki & M. Vergassola, *Rev. Mod. Phys.* **73**, 913 (2001)



<u>Surrogate Gaussian velocity field</u>

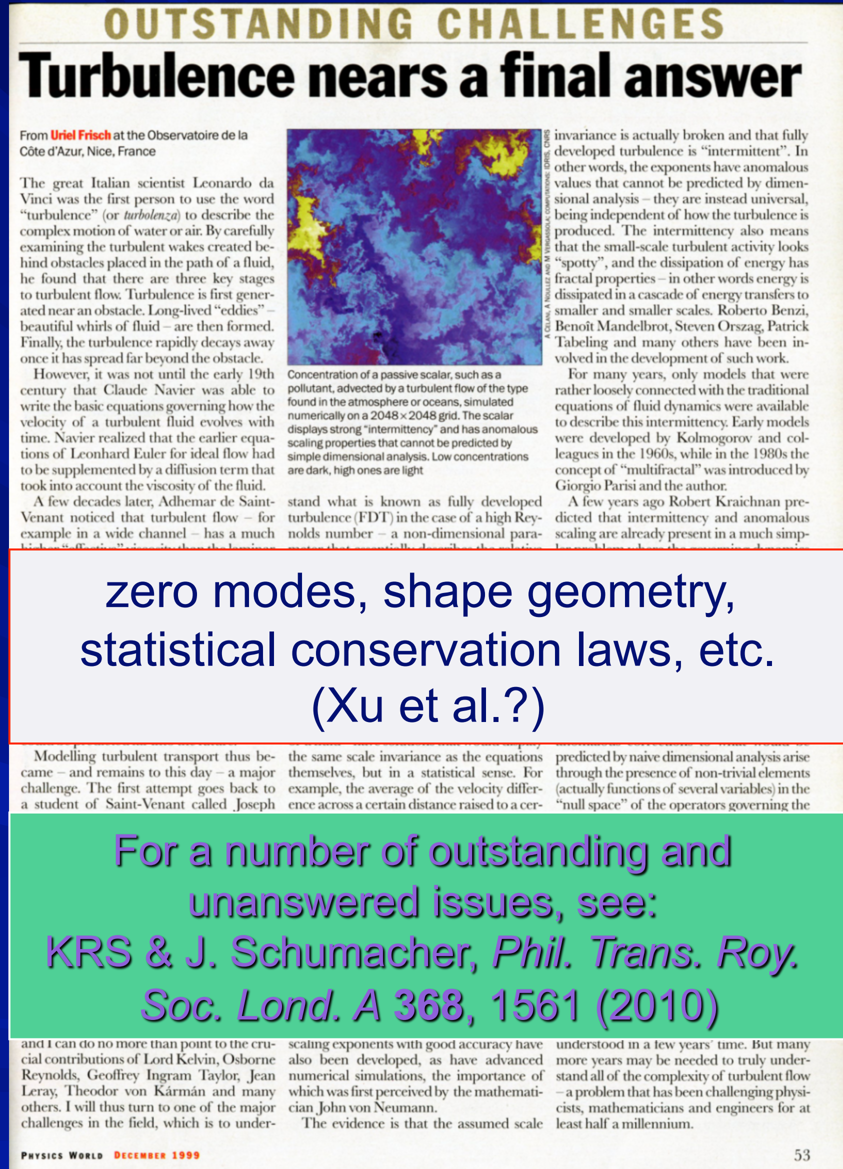
$\langle u_i(\mathbf{x};t)u_j(\mathbf{y};t')\rangle = |\mathbf{x}-\mathbf{y}|^{2-\gamma}\,\delta(t-t')$

$\gamma = 2/3$ recovers Richardson's diffusion

Forcing for stationarity:

$\langle f_\theta(\mathbf{x};t)f_\theta(\mathbf{y};t')\rangle = C(r/L)\,\delta(t-t')$

C(r/L) is non-zero only on the large scale, decays rapidly to zero for smaller scale.



OUTSTANDING CHALLENGES

Turbulence nears a final answer

From Uriel Frisch at the Observatoire de la Côte d'Azur, Nice, France

The great Italian scientist Leonardo da Vinci was the first person to use the word "turbulence" (or *turbolenza*) to describe the complex motion of water or air. By carefully examining the turbulent wakes created behind obstacles placed in the path of a fluid, he found that there are three key stages to turbulent flow. Turbulence is first generated near an obstacle. Long-lived "eddies" – beautiful whirls of fluid – are then formed. Finally, the turbulence rapidly decays away once it has spread far beyond the obstacle.

However, it was not until the early 19th century that Claude Navier was able to write the basic equations governing how the velocity of a turbulent fluid evolves with time. Navier realized that the earlier equations of Leonhard Euler for ideal flow had to be supplemented by a diffusion term that took into account the viscosity of the fluid.

A few decades later, Adhemar de Saint-Venant noticed that turbulent flow – for example in a wide channel – has a much

Concentration of a passive scalar, such as a pollutant, advected by a turbulent flow of the type found in the atmosphere or oceans, simulated numerically on a 2048×2048 grid. The scalar displays strong "intermittency" and has anomalous scaling properties that cannot be predicted by simple dimensional analysis. Low concentrations are dark, high ones are light

A CELANI, A NOULLEZ AND M VERGASSOLA; COMPUTATIONS: IDRIS, CNRS

stand what is known as fully developed turbulence (FDT) in the case of a high Reynolds number – a non-dimensional para-

invariance is actually broken and that fully developed turbulence is "intermittent". In other words, the exponents have anomalous values that cannot be predicted by dimensional analysis – they are instead universal, being independent of how the turbulence is produced. The intermittency also means that the small-scale turbulent activity looks "spotty", and the dissipation of energy has fractal properties – in other words energy is dissipated in a cascade of energy transfers to smaller and smaller scales. Roberto Benzi, Benoît Mandelbrot, Steven Orszag, Patrick Tabeling and many others have been involved in the development of such work.

For many years, only models that were rather loosely connected with the traditional equations of fluid dynamics were available to describe this intermittency. Early models were developed by Kolmogorov and colleagues in the 1960s, while in the 1980s the concept of "multifractal" was introduced by Giorgio Parisi and the author.

A few years ago Robert Kraichnan predicted that intermittency and anomalous scaling are already present in a much simp-

Modelling turbulent transport thus became – and remains to this day – a major challenge. The first attempt goes back to a student of Saint-Venant called Joseph

and I can do no more than point to the crucial contributions of Lord Kelvin, Osborne Reynolds, Geoffrey Ingram Taylor, Jean Leray, Theodor von Kármán and many others. I will thus turn to one of the major challenges in the field, which is to under-

the same scale invariance as the equations themselves, but in a statistical sense. For example, the average of the velocity difference across a certain distance raised to a cer-

scaling exponents with good accuracy have also been developed, as have advanced numerical simulations, the importance of which was first perceived by the mathematician John von Neumann.

The evidence is that the assumed scale

predicted by naive dimensional analysis arise through the presence of non-trivial elements (actually functions of several variables) in the "null space" of the operators governing the

understood in a few years' time. But many more years may be needed to truly understand all of the complexity of turbulent flow – a problem that has been challenging physicists, mathematicians and engineers for at least half a millennium.

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zero modes, shape geometry, statistical conservation laws, etc. (Xu et al.?)

For a number of outstanding and unanswered issues, see: KRS & J. Schumacher, *Phil. Trans. Roy. Soc. Lond. A* **368**, 1561 (2010)

$\langle \Delta_r u^2 \rangle \sim r^{\zeta_2}$

Standard "theory" gets the ζ_2 by assuming that the structure functions obey the same symmetries as the equations. Two questions arise:

1. In $\langle \Delta_r u^4 \rangle \sim r^{\zeta_4}$

the same argument yields $\zeta_4 = 2\zeta_2$ (in general, $\zeta_{2n} = n\zeta_2$)

E.g., flatness = $\langle \Delta_r u^4 \rangle / \langle \Delta_r u^2 \rangle^2$ = constant.

Measurements have shown that the flatness $\rightarrow \infty$ as $r \rightarrow 0$

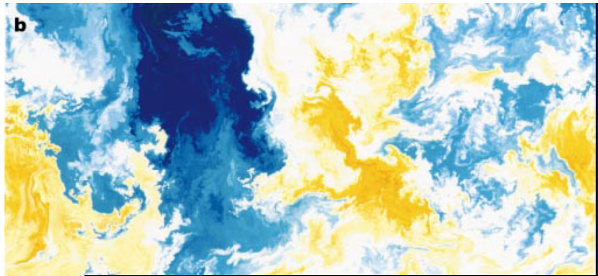
[i.e., $\zeta_4 < 2\zeta_2$ (or generally $\zeta_{2n} < n\zeta_2$)]

The exponent of any given order order has to be determined on its own merit.

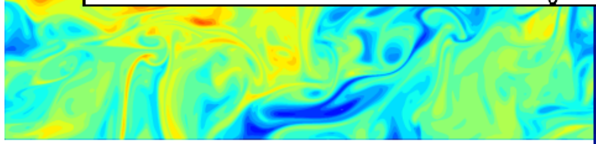
"Anomalous exponents"

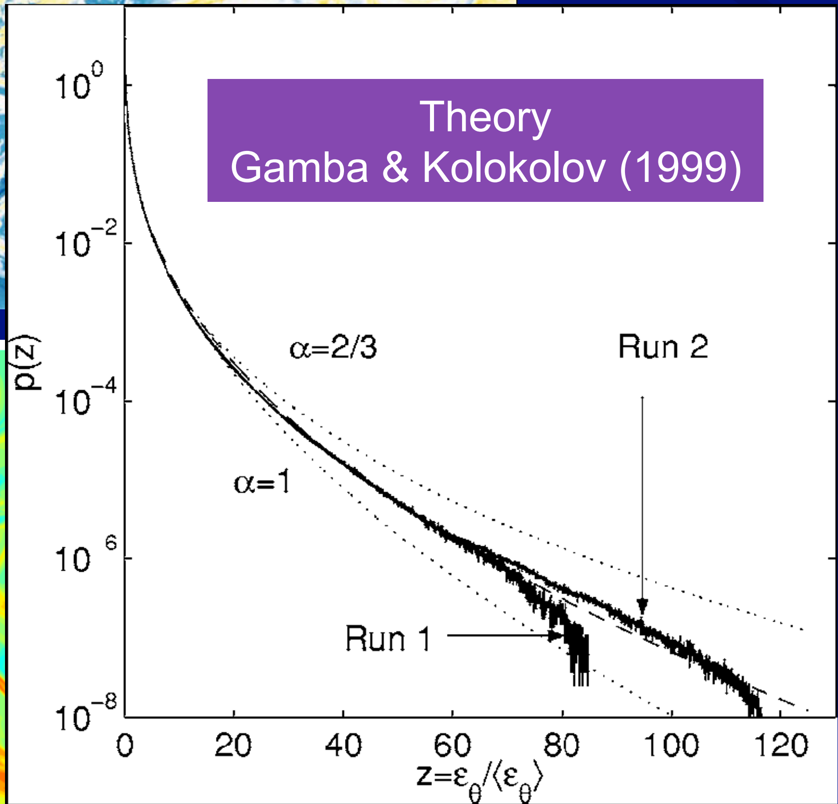
2. In the inertial range, we have $\langle \Delta_r u^3 \rangle = -4/5 \langle \varepsilon \rangle r$

Breaking of symmetry. Are there are other statistical conservation laws whose symmetry breaking provides the basis for determining the exponents of higher orders.



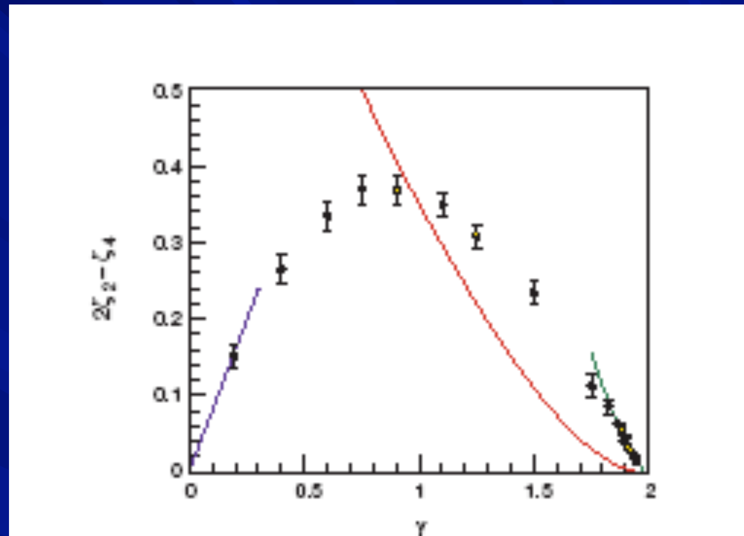
γ= 0.5,
8192^2
S. Chen





Gotoh & Watanabe

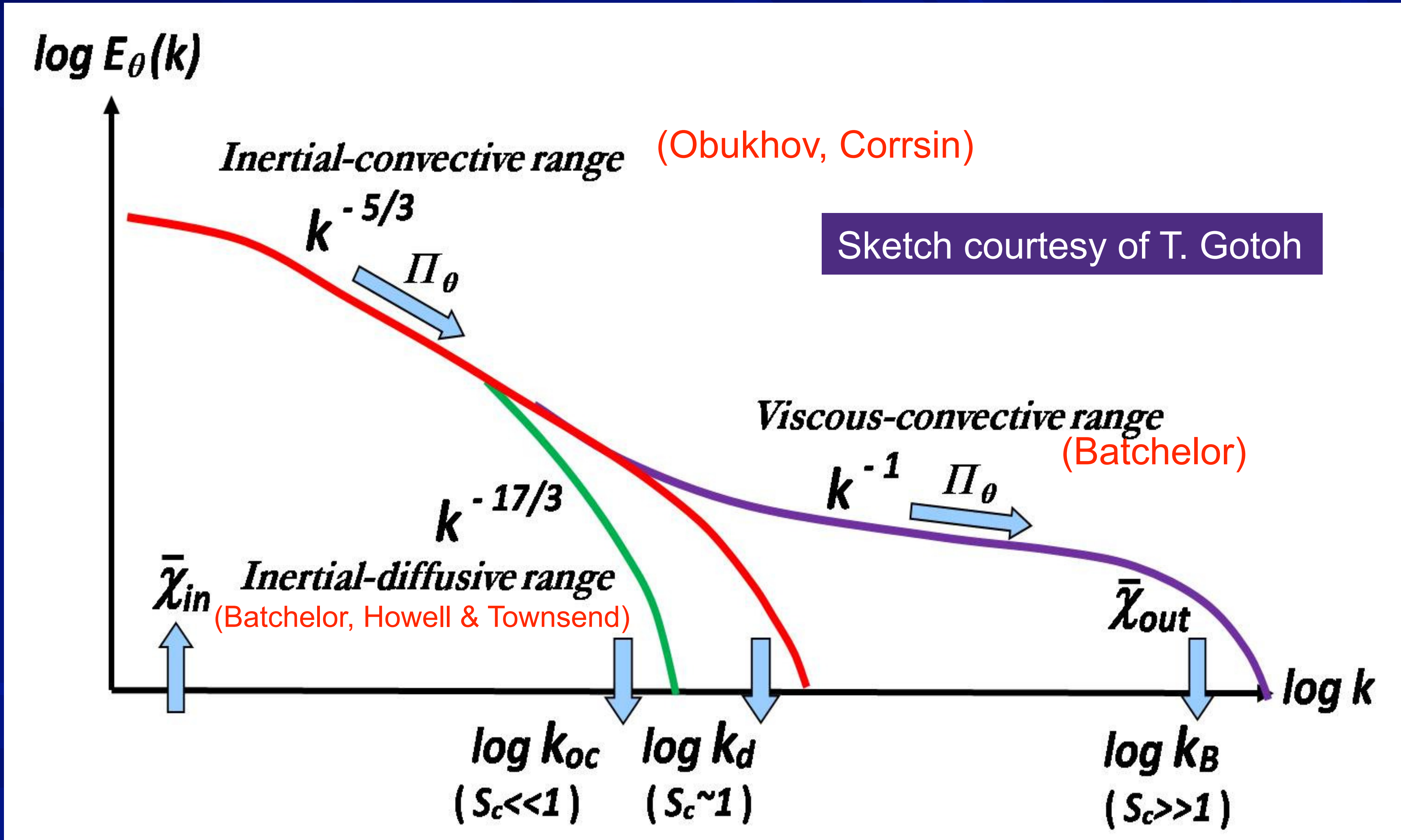
$2\zeta_2 - \zeta_4$



A measure of anomalous scaling, $2\zeta_2 - \zeta_4$, versus the index γ, for the Kraichnan model. The circles are obtained from Lagrangian Monte Carlo simulations (from U. Frisch's group). The results are compared with analytic perturbation theories (blue, green) and an ansatz due to Kraichnan (red).

Mixing process itself imprints features independent of the velocity field!

The passive scalar spectrum
What we know
(and what we don’t)

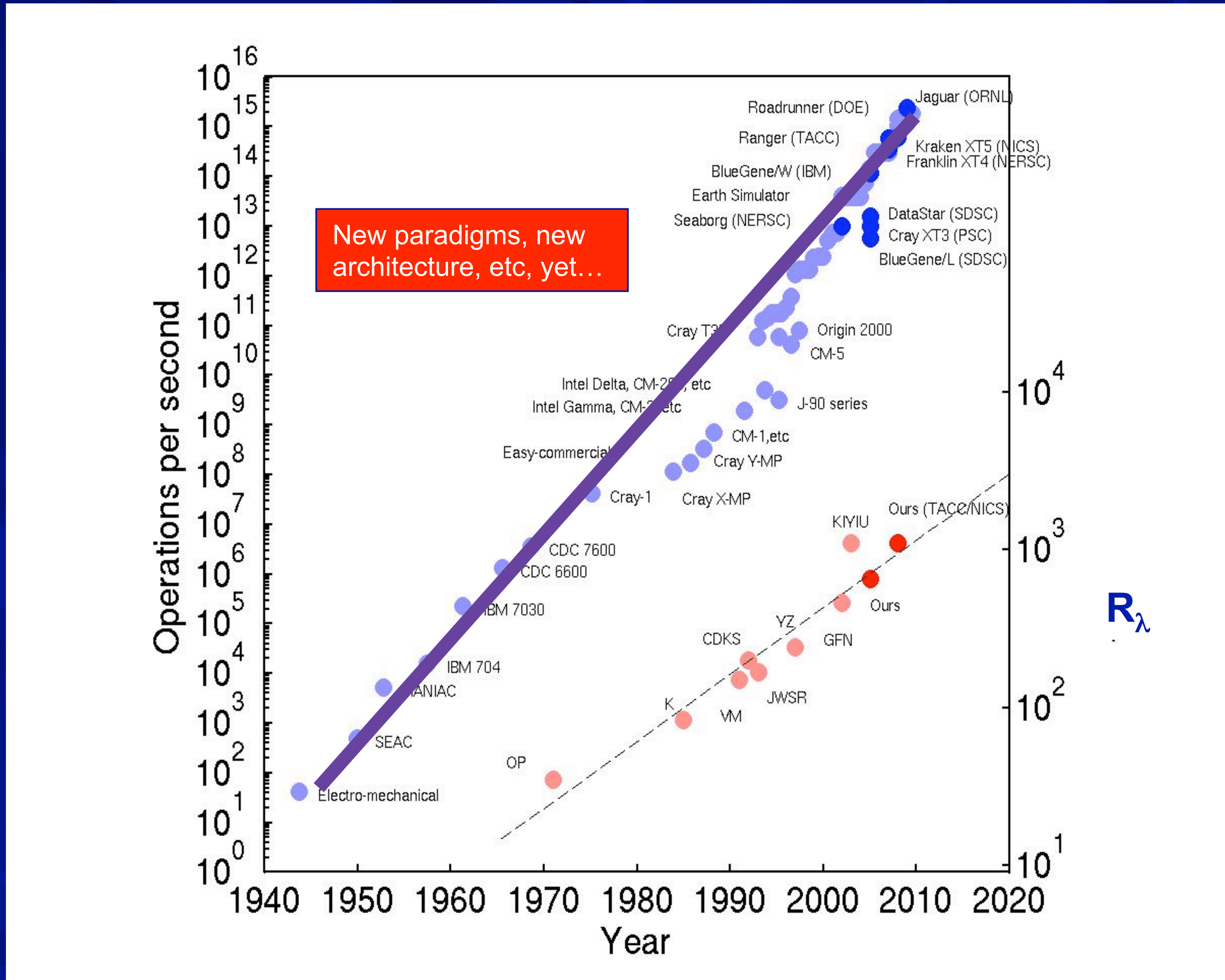


log Eθ(k)
Inertial-convective range
(Obukhov, Corrsin)
k - 5/3
Πθ
Sketch courtesy of T. Gotoh
Viscous-convective range
(Batchelor)
k - 1
Πθ
k - 17/3
χ̄in
Inertial-diffusive range
(Batchelor, Howell & Townsend)
χ̄out
log k
log koc
(Sc<<1)
log kd
(Sc~1)
log kB
(Sc>>1)

P.K. Yeung
Georgia Tech.

Diego Donzis
Texas A&M

Jörg Schumacher
TU Ilmenau



Massive parallelism, up to O(10^5) CPU cores, so doing simulations has become a big task in itself.

Sc $\lesssim$ 1: Obukhov-Corrsin scaling

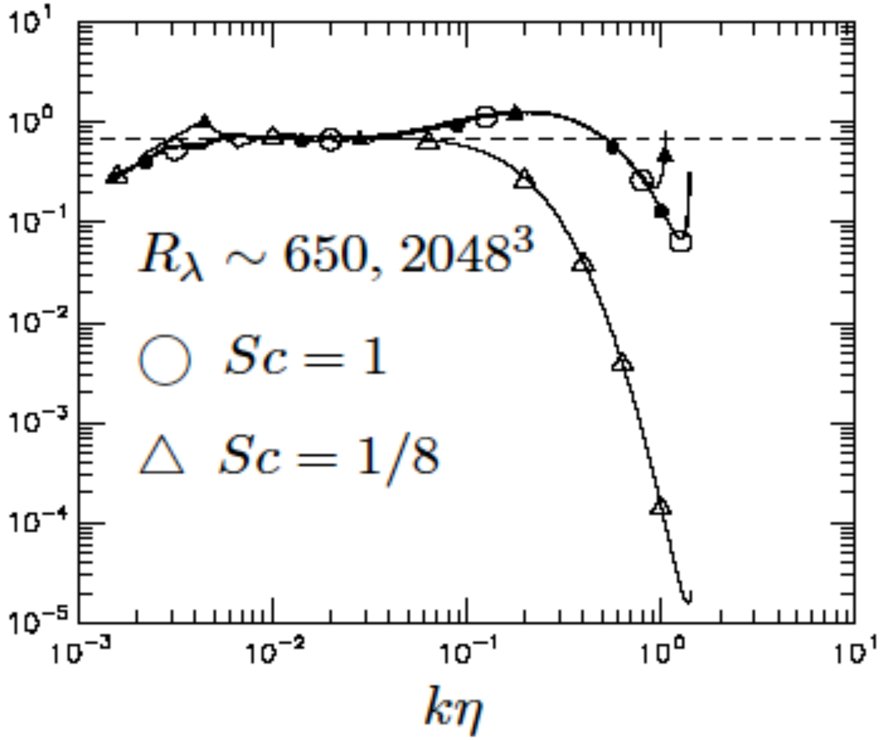
- Inertial-convective:

  $$E_\phi(k) \sim \langle\chi\rangle\langle\epsilon\rangle^{-1/3}k^{-5/3}$$

  (for $1/L \ll k \ll 1/\eta_{OC}$)

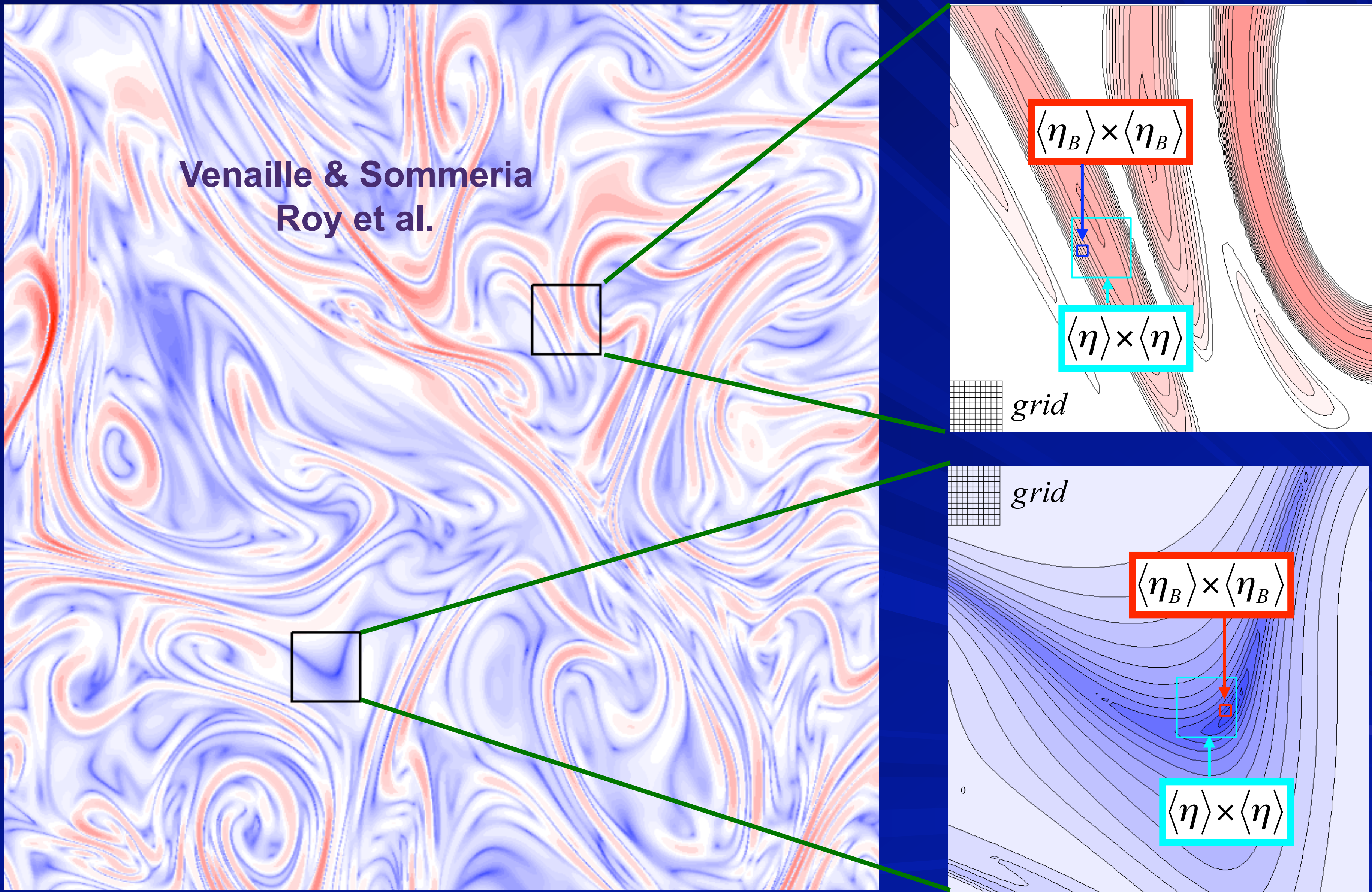
- Yeung *et al.* PoF 2005:
  - $C_{OC} \approx 0.67$ in 3D spectrum, consistent with survey of experiments (Sreenivasan PoF 1996)
  - "bottleneck" apparent for $Sc = 1$ (or precursor to k^{-1} for $Sc > 1$)

Compensated spectra

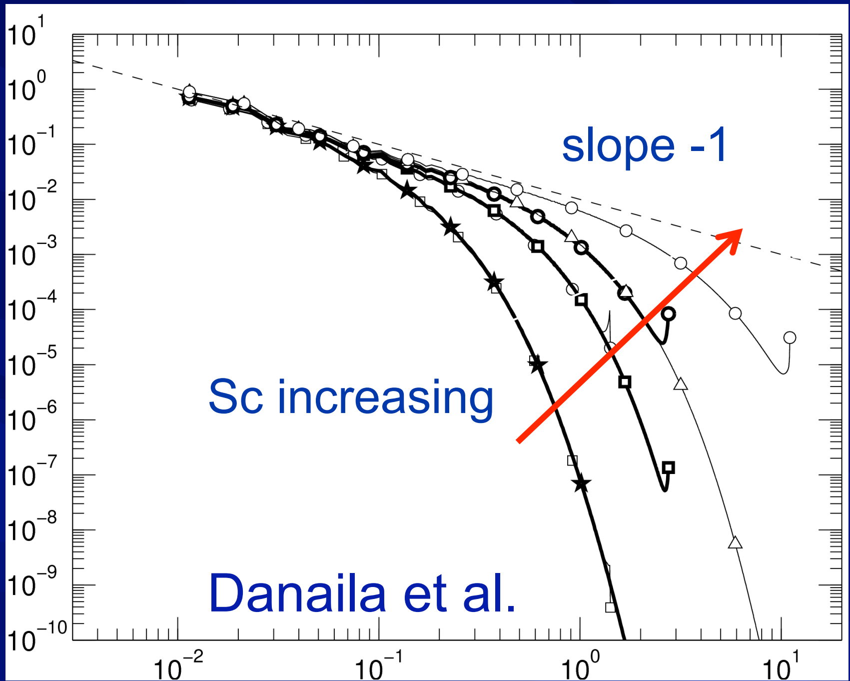


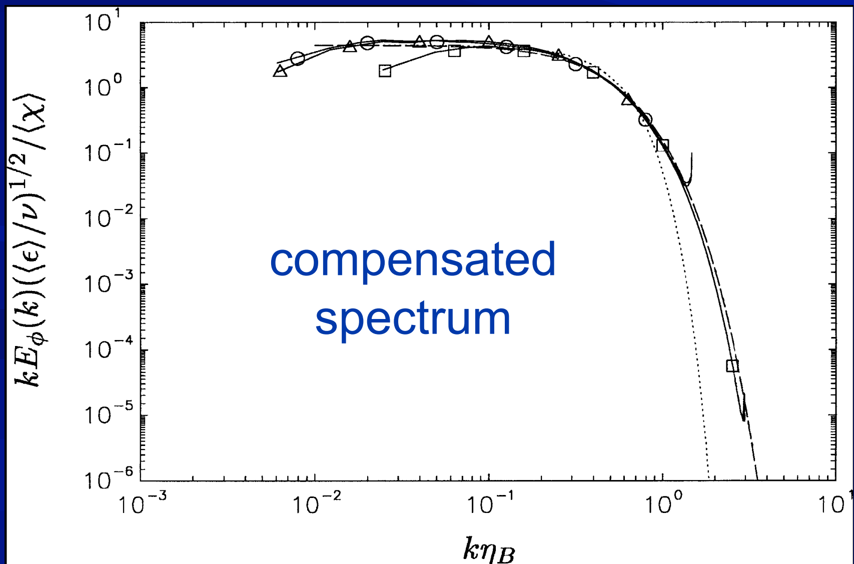
Consistent with isotropic random forcing of scalars (Watanabe & Gotoh 2004, 2007; ▲, •)

For large Sc, computational domain size scales as $Re^3\ Sc^2$.



The viscous convective region





Reynolds number: Re >>1

Schmidt number, Sc = ν/κ >>1

In support of the -1 power law

Gibson & Schwarz, *JFM* **16**, 365 (1963)

KRS & Prasad, *Physica D* **38**, 322 (1989)

Expressing doubts

Miller & Dimotakis, *JFM* **308**, 129 (1996)

Williams et al. *Phys. Fluids* **9**, 2061 (1997)

Simulations in support

Holzer & Siggia, *Phys. Fluids* **6**, 1820 (1994)

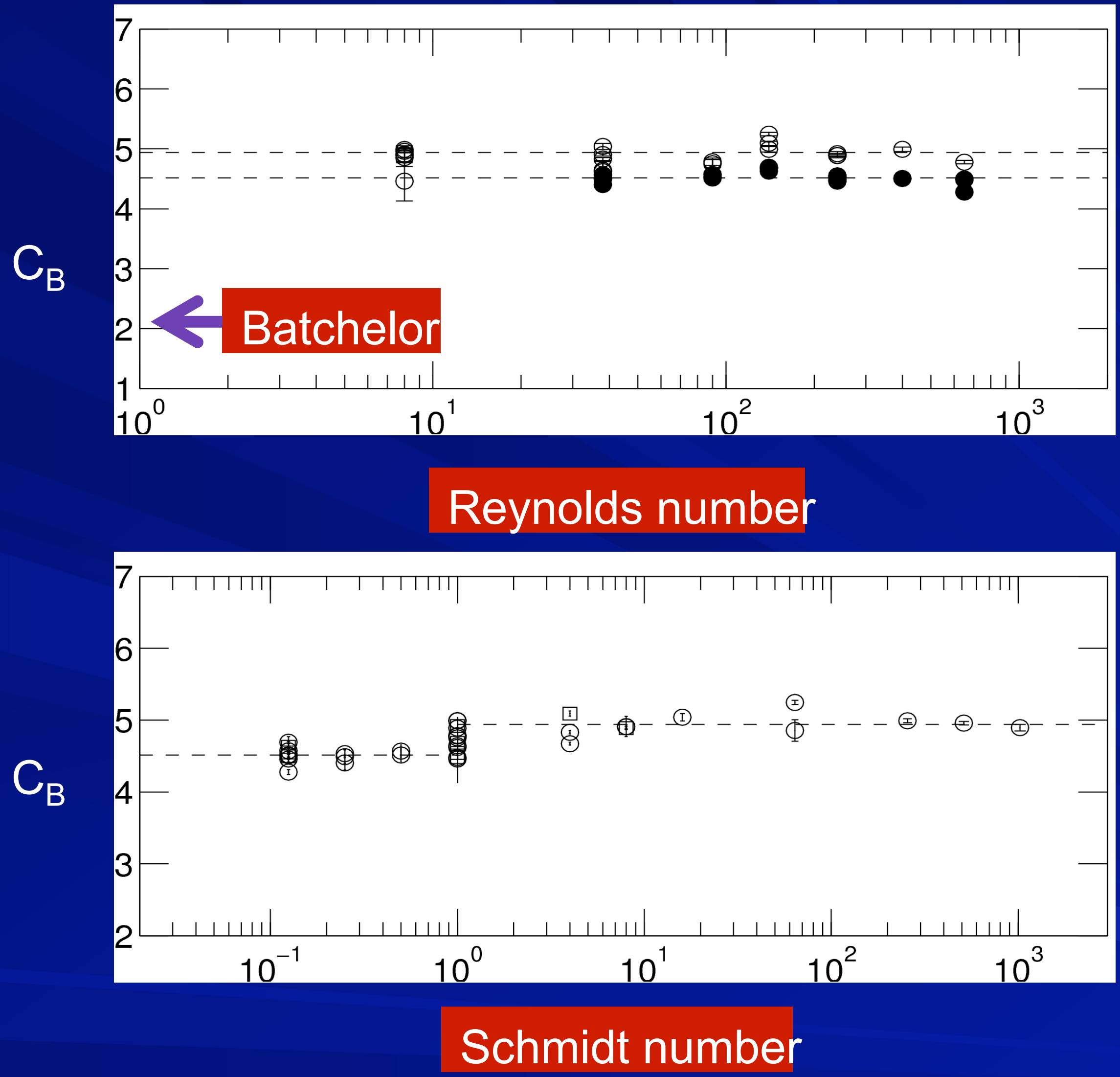
Batchelor (1956)

$E_\theta(k) = C_B\, \kappa(\nu/\varepsilon)^{1/2}k^{-1}\exp[-q(k\eta_B)^2]$

Kraichnan (1968)

$E_\theta(k) = C_B\, \kappa(\nu/\varepsilon)^{1/2}k^{-1}\,[1+(6q)^{1/2}k\eta_B \times$

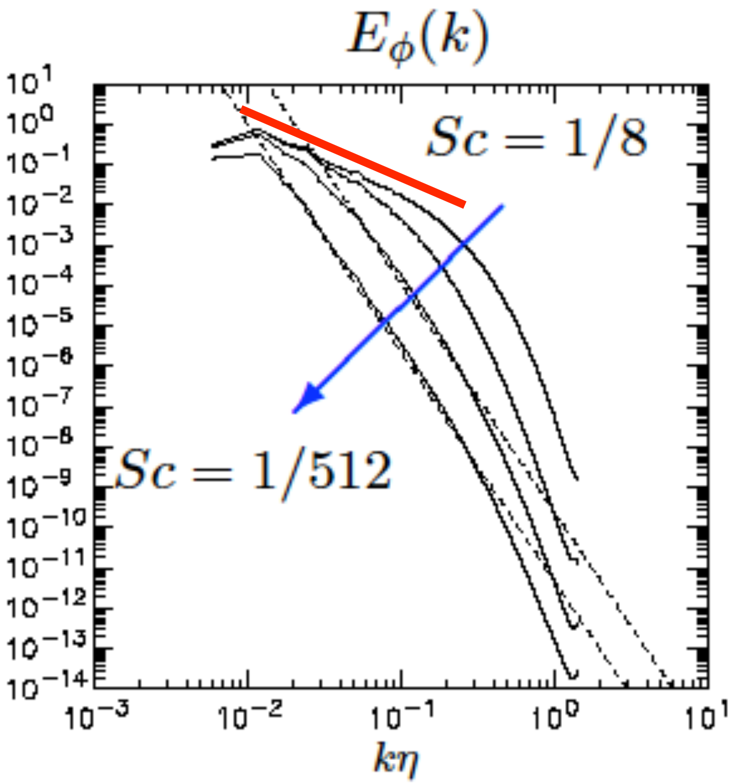
$\exp(-(6q)^{1/2}k\eta_B)]$



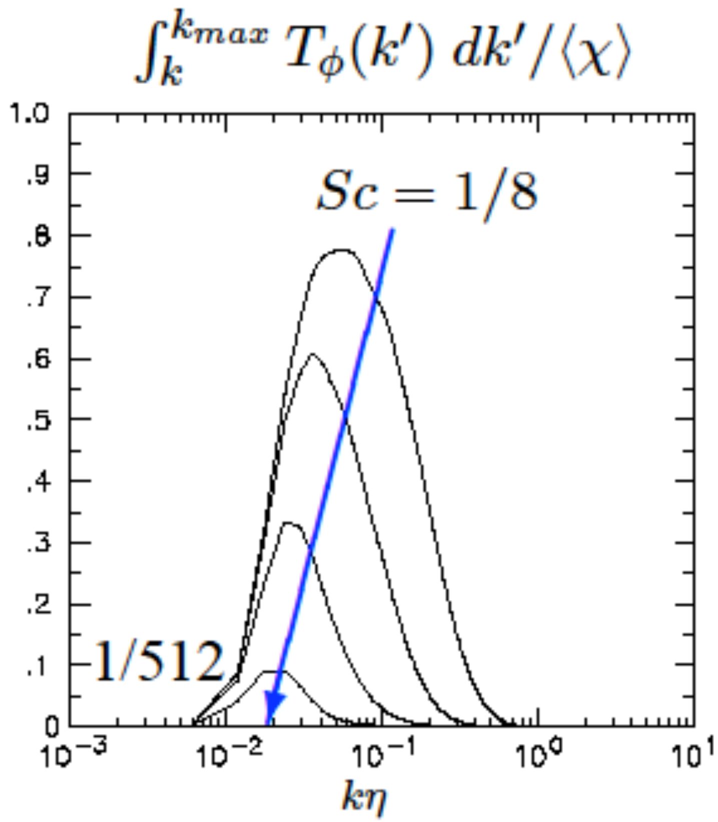
Donzis, KRS & P.K. Yeung, *Flow, Turbulence and Combustion* **85**, 549 (2010)

DNS results for $\mathrm{Sc} \ll 1$

512^3, $R_\lambda \approx 150$, $(4\pi)^3$ domain



Supports $k^{-17/3}$ spectrum



Transfer flux is weak

- Higher *Re* simulations planned, possibly yet-lower Schmidt nos.

The Yaglom relation (1949)

$$<\Delta_r u\ (\Delta_r \theta)^2> = -(2/3)<\chi>r$$

G. Stolovitzky, P. Kailasnath & KRS, JFM 297, 275 (1995)

- Refined similarity hypothesis

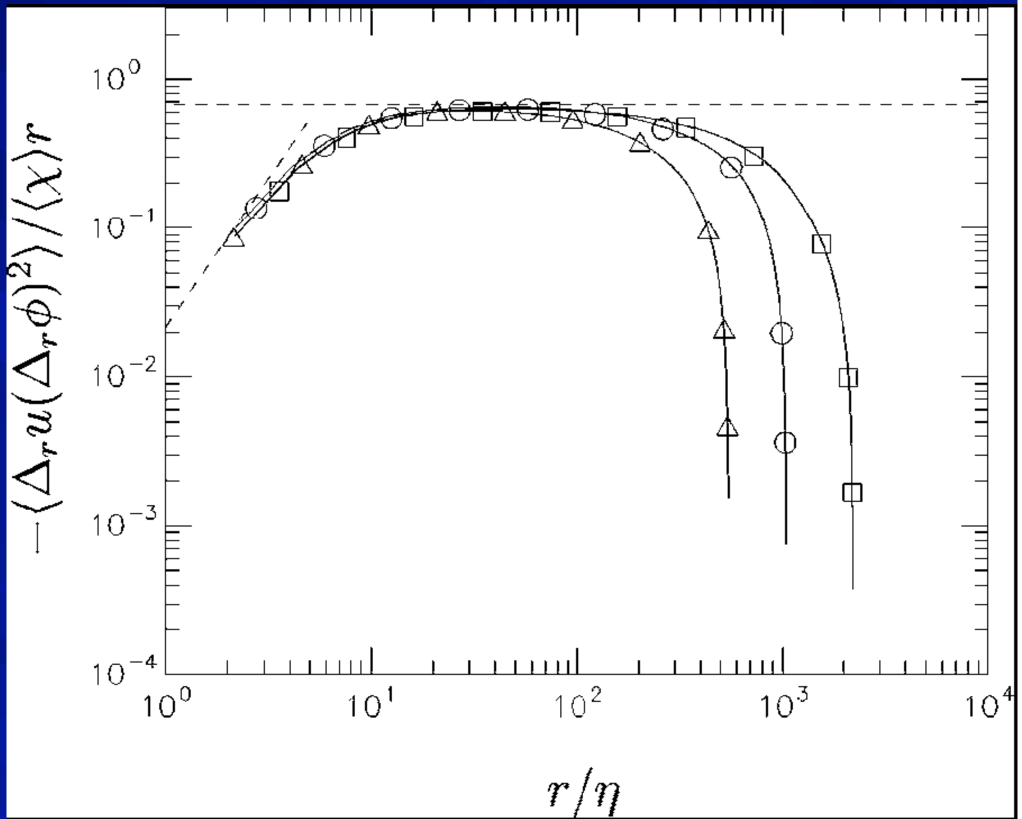
L. Danaila, F. Anselmet, T. Zhou & R.A. Antonia, JFM 391, 359 (1999)

- Extension to non-stationary forcing conditions

P. Orlandi & R.A. Antonia, JFM 451, 99 (2002): DNS

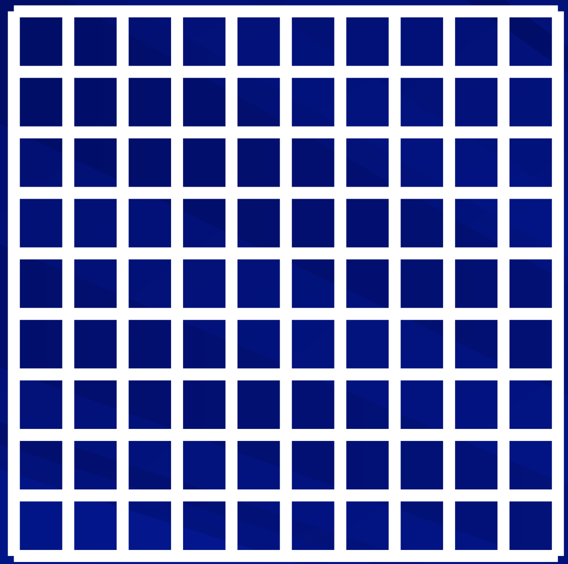
L. Midlarsky, JFM 475, 173 (2003): Experiment

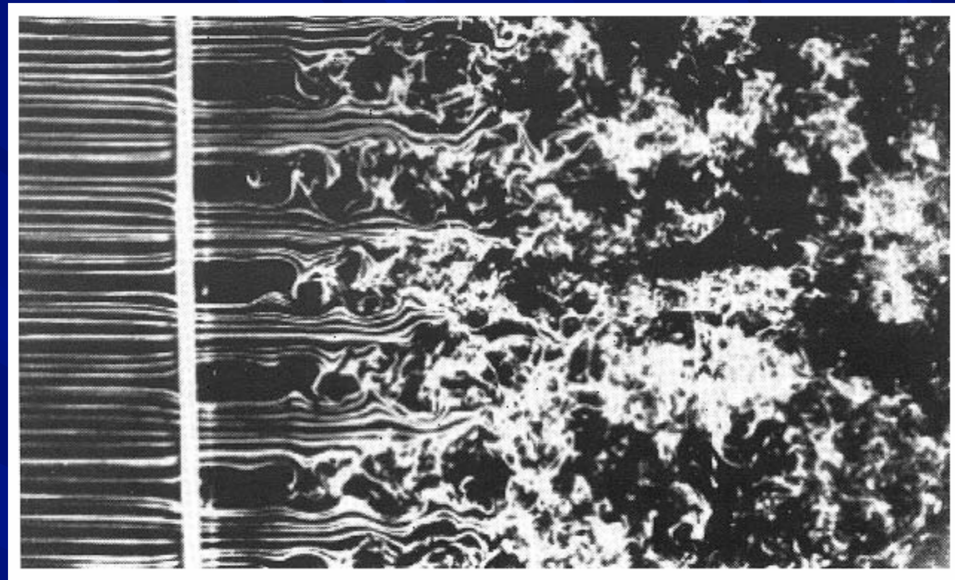
- Conditions of Reynolds and Peclet numbers under which the Yagolm equation holds



Some large scale features

Decaying fields of turbulence and scalar



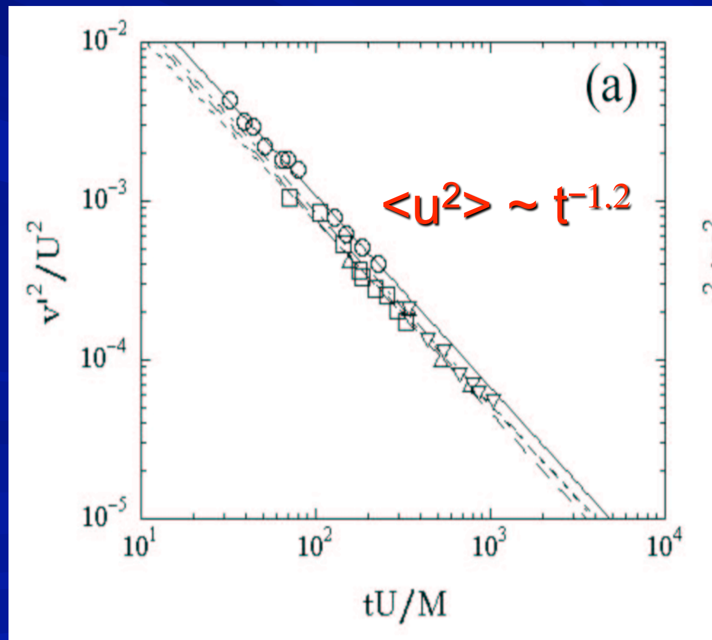


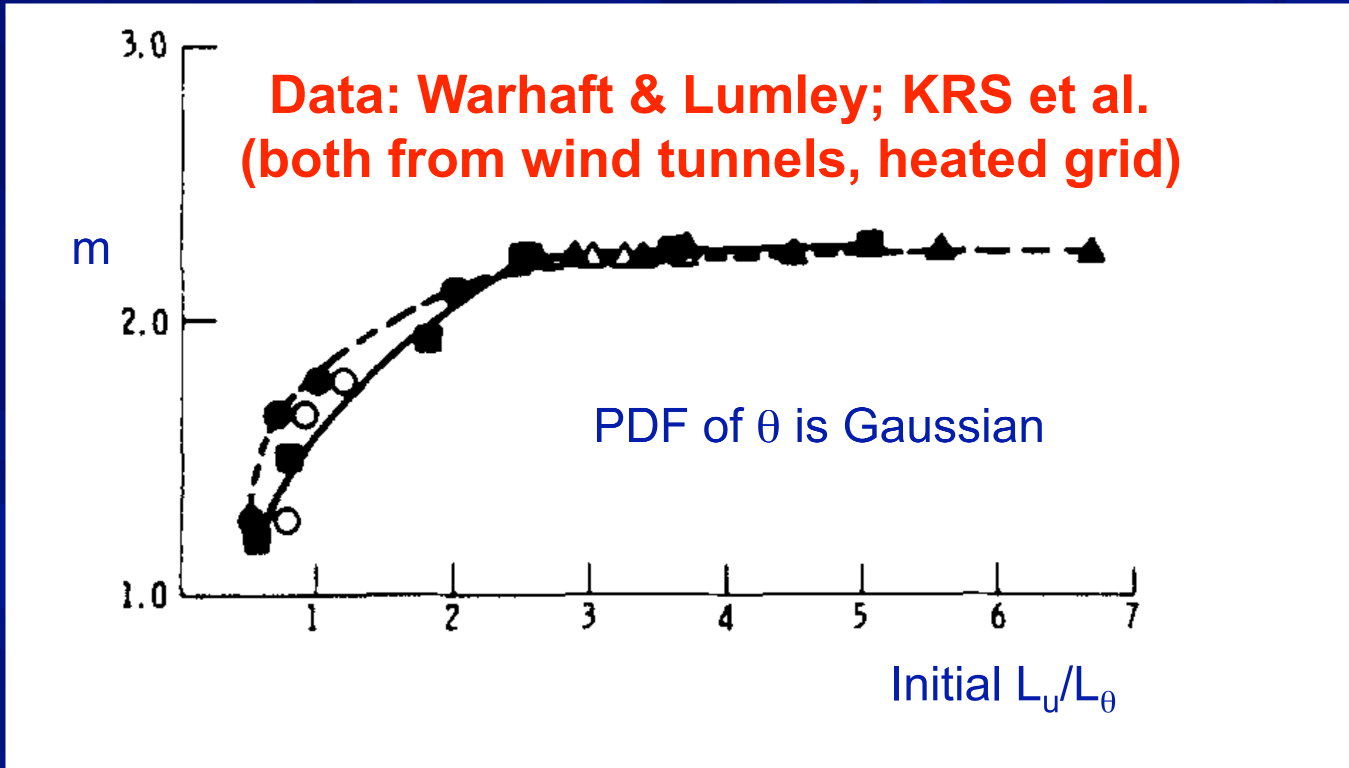
- L_u is set by the mesh size
- L_θ can be set independently and L_u/L_θ can be varied
- Diffusivity of the scalar (i.e., *Pr* or *Sc* = ν/κ) is a variable.

$\langle\theta^2\rangle \sim t^{-m}$ (variable m)

$m - m_0 = f(Re; Sc; L_u/L_\theta)$?

m_0: asymptotic m for large values of the arguments

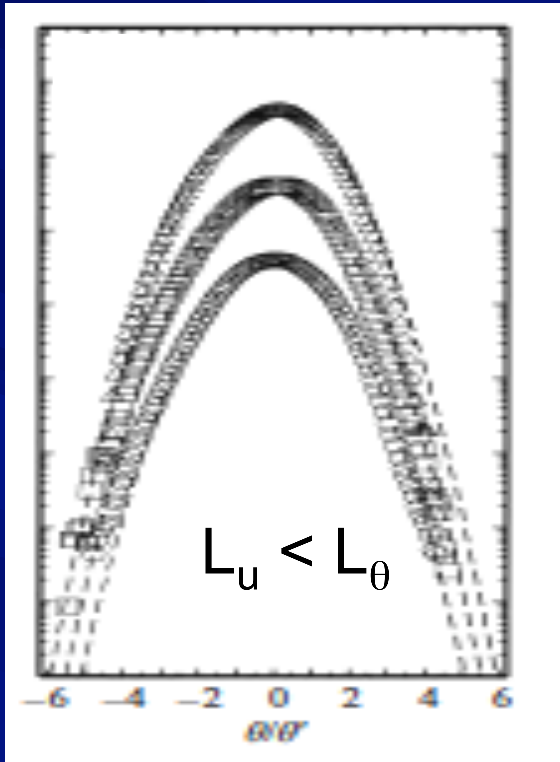


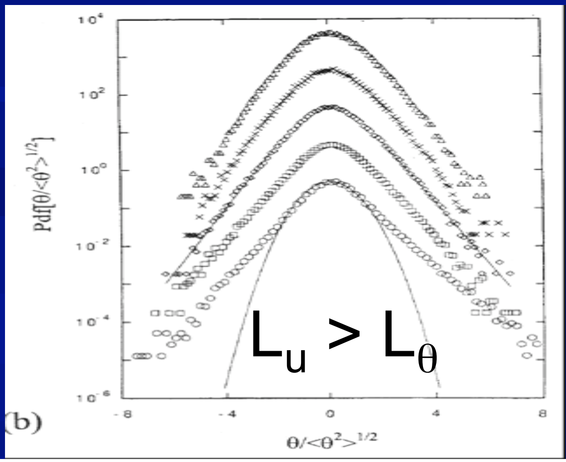


Non-uniqueness of the exponent is not difficult to understand qualitatively but difficult to make a theory for.

Durbin, Phys. Fluids 25, 1328 (1982)

Effect of length-scale ratio: PDF of θ in stationary turbulence





Both PDFs are for stationary velocity and scalar fields, under comparable Reynolds and Schmidt numbers.

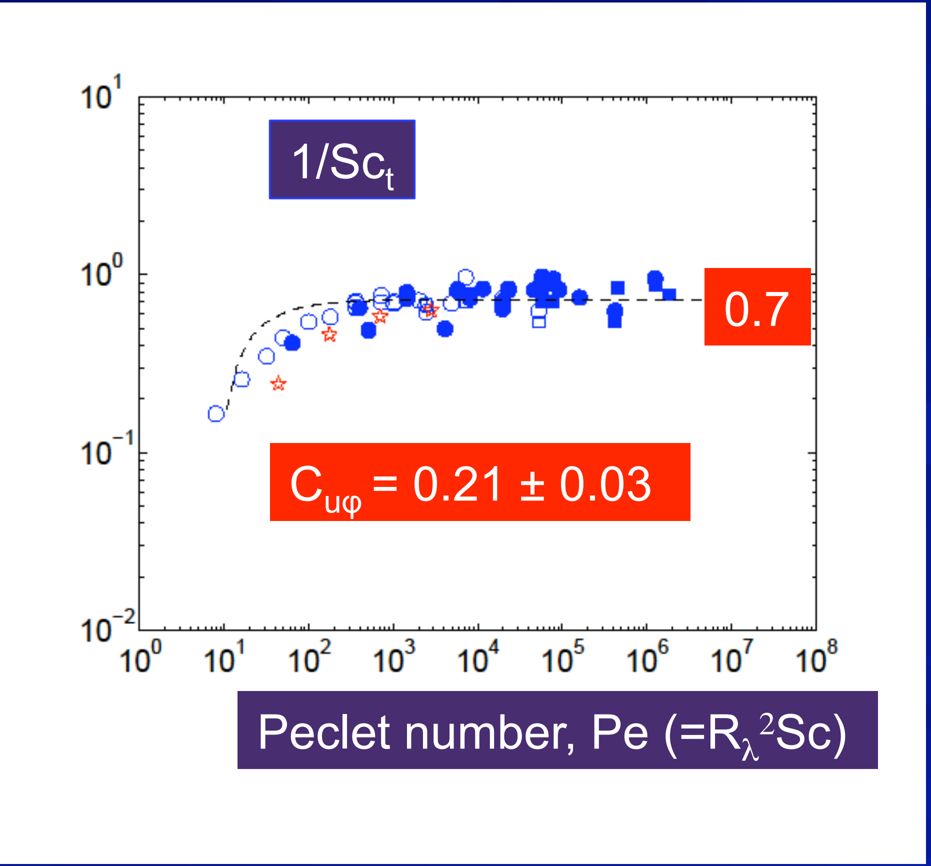
Passive scalars in homogeneous flows most often have Gaussian tails, but long tails are observed for column-integrated tracer distributions in horizontally homogeneous atmospheres.

Models of Bourlioux & Majda, *Phys. Fluids* **14**, 881 (2002), closely connected with models studied by Avellaneda & Majda

Probability density function of the passive scalar
Top: Ferchichi & Tavoularis (2002)
Bottom: Warhaft (2000)

Direct Numerical Simulations
(P.K. Yeung, D. Donzis, KRS)

$8 < R_\lambda < 650$
$1/512 < Sc < 1024$
Different forcing schemes



Experiment

Homogeneous shear flows
Boundary layers
Jets
Wakes

Dimensional Theory

Flux spectrum

$$E_{u\varphi}(k) = C_{u\varphi} G \langle\varepsilon\rangle^{1/3} k^{-7/3}$$

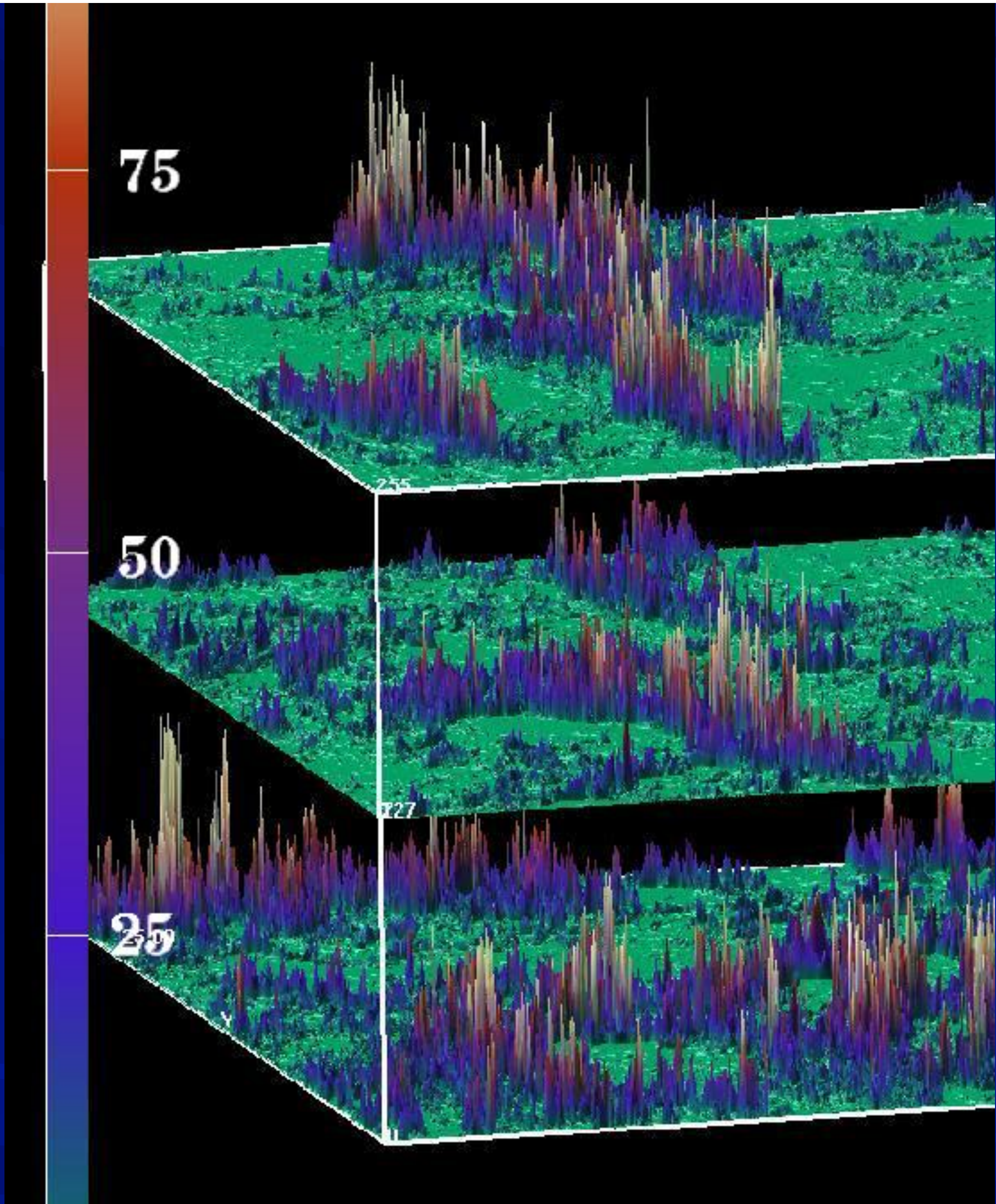
in the inertial convection range
(Lumely 1964)

Using $\langle u\varphi \rangle = -\int E_{u\varphi}(k)\, dk$ (with appropriate limits),

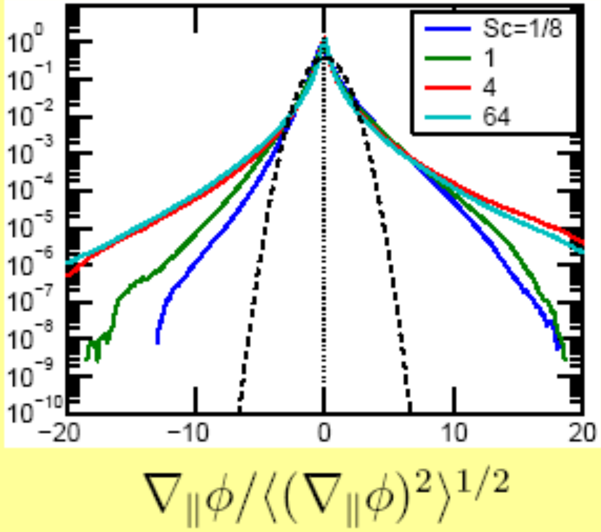
we get

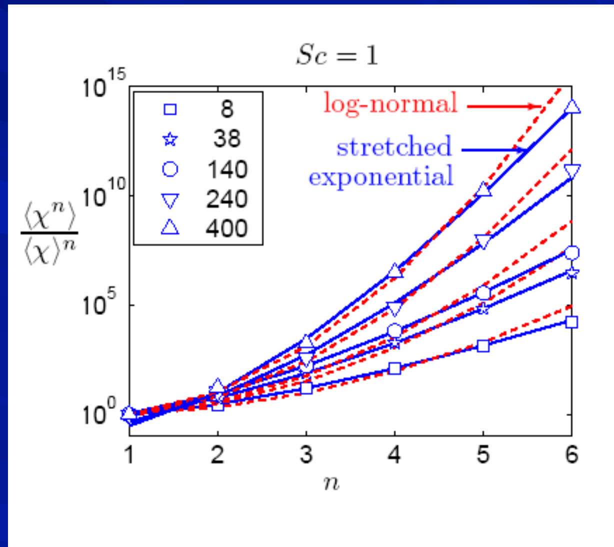
$$1/Sc_t = (10/3)\, C_{u\varphi} (1 - 1/Pe)$$

Doering & Thiffeault

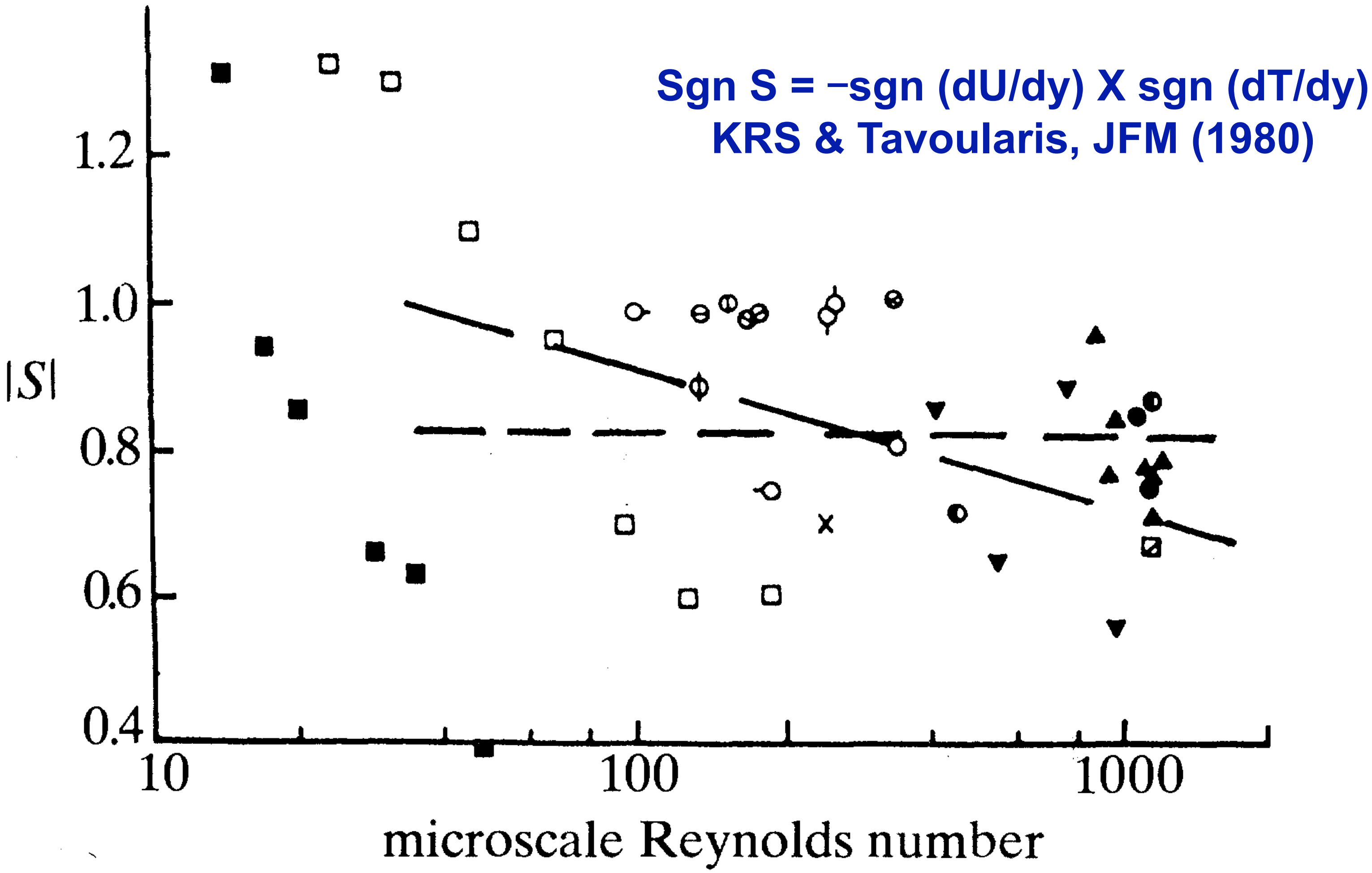


Dissipation intermittency (Dissipative anomaly holds)





Anisotropy of small scales (with R.A. Antonia)



Some consequences of fluctuations

0. Traditional definitions

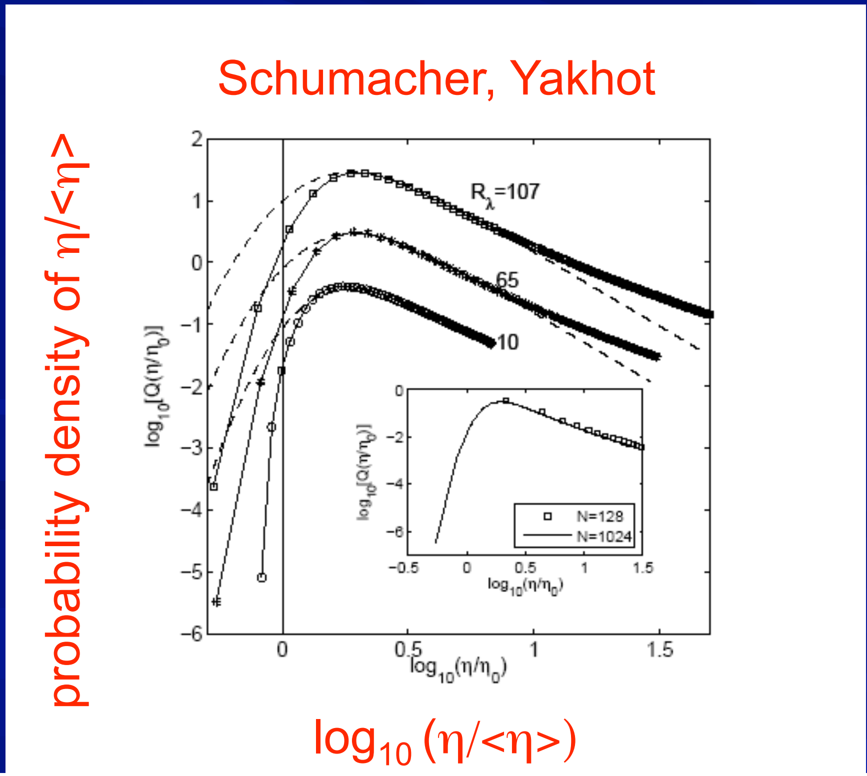
$\langle\eta\rangle = (\nu^3/\langle\varepsilon\rangle)^{1/4}$, $\langle\eta_B\rangle = \langle\eta\rangle/Sc^{1/2}$, $\langle\tau_d\rangle = \langle\eta_B\rangle^2/\kappa$

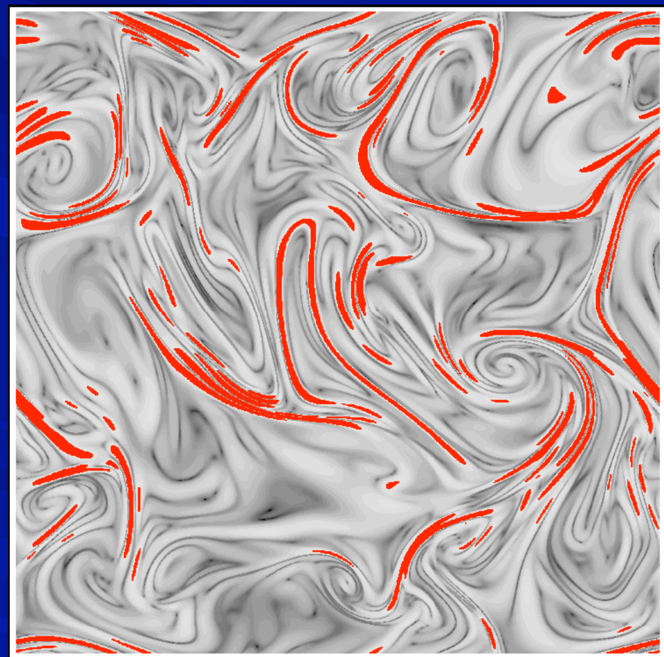
1. Local scales

$\eta = (\nu^3/\varepsilon)^{1/4}$, or define η through $\eta\delta_\eta u/\nu = 1$

$\eta_B = \eta/Sc^{1/2,}$, $\tau_d = \eta_B{}^2/\kappa$

2. Distribution of length scales





3. The velocity field is analytic only in the range $r < \eta$ (and the scalar field only for $r < \eta_B$)

4. Minimum length scale $\eta_{min} = <\eta> Re^{-1/4}$ (Schumacher, KRS and Yakhot 2007)

5. Average diffusion time scale $<\tau_d> = <\eta_B^2>/\kappa$, not $<\tau_d> = <\eta_B>^2/\kappa$

6. From the distribution of length scales, we have $<\tau_d> = <\eta_B^2>/\kappa \approx 10 <\eta_B>^2/\kappa$

7. Eddy diffusive time/molecular diffusive time $\approx Re^{1/2}/100$;exceeds unity only for $Re \approx 10^4$ (mixing transition advocated by Dimotakis, short-circuit in cascades of Villermaux, etc)

Other mixing problems

- Passive mixing under differential diffusion
  - J.R. Saylor & KRS, *Phys. Fluids* **10**, 1135 (1998)
- Mixing of fluids of different densities, where the mixing has a large influence on the velocity field (e.g., thermal convection, Rayleigh-Taylor instability, radiation effects)
- Those accompanied by changes in composition, density, enthalpy, pressure, etc. (e.g., combustion, detonation, supernova)
  - N. Peters (later in this session)

Whither universality of small-scale scalar?

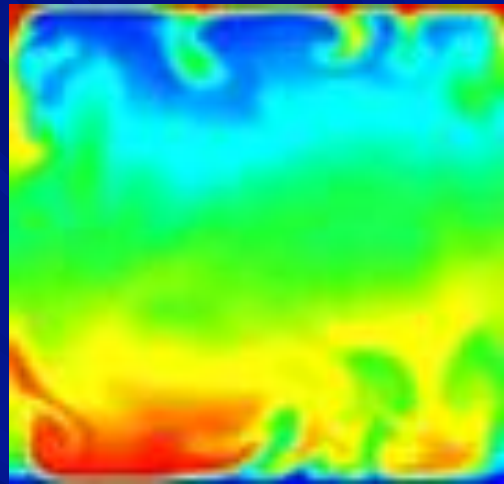
Active scalars

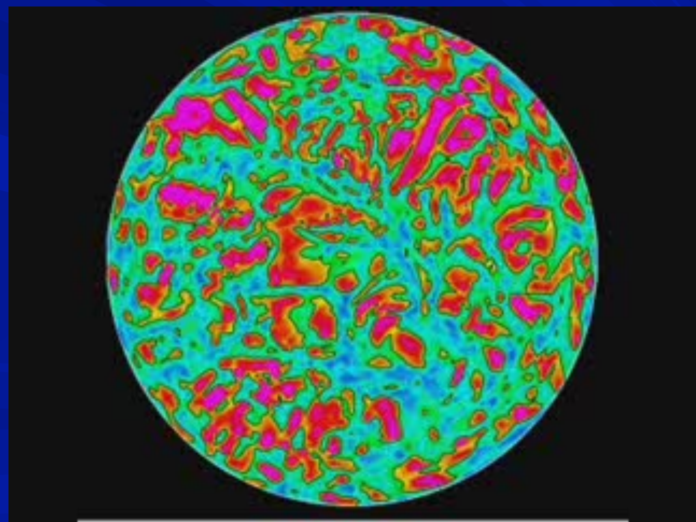
$$\partial_t a = \mathbf{u}.\nabla a + \kappa\Delta a + \mathbf{F}_a$$

$$u_i(\mathbf{x};t) = \int d\mathbf{y}\, K_i(\mathbf{x},\mathbf{y})\, a(\mathbf{y},t)$$

Simple case: Boussinesq approximation

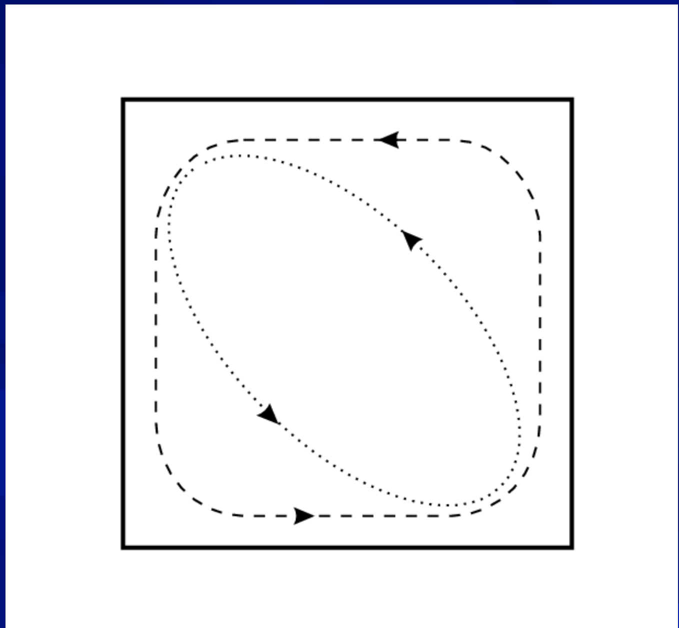
$$NS = -\beta g a$$

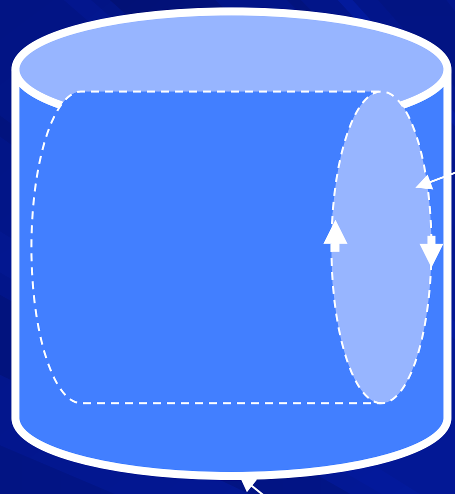


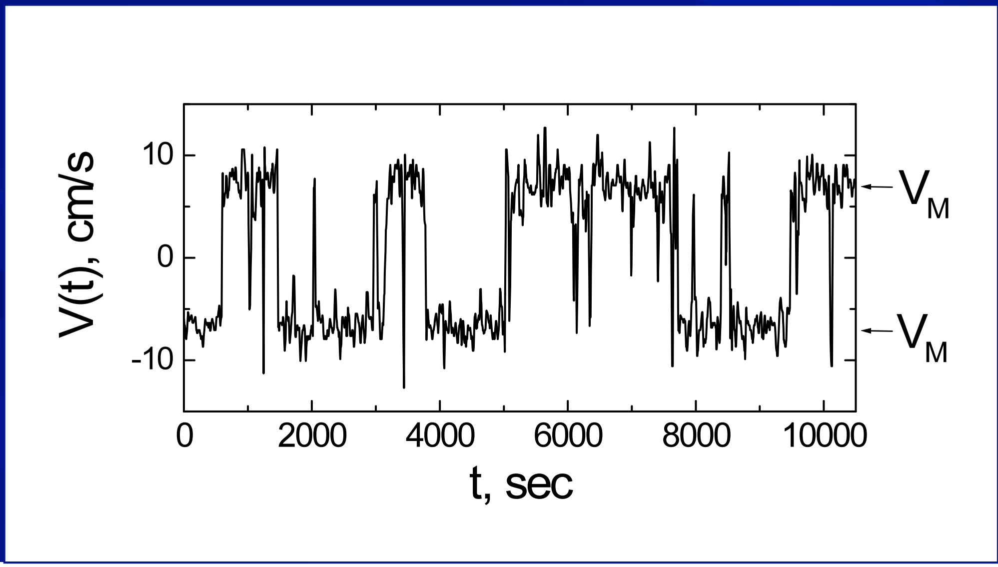


The mean wind

The “mean wind” breaks symmetry, with its own consequences







Segment of 120 hr record

Niemela, KRS, Donnelly (2002)

How are the reversals distributed?

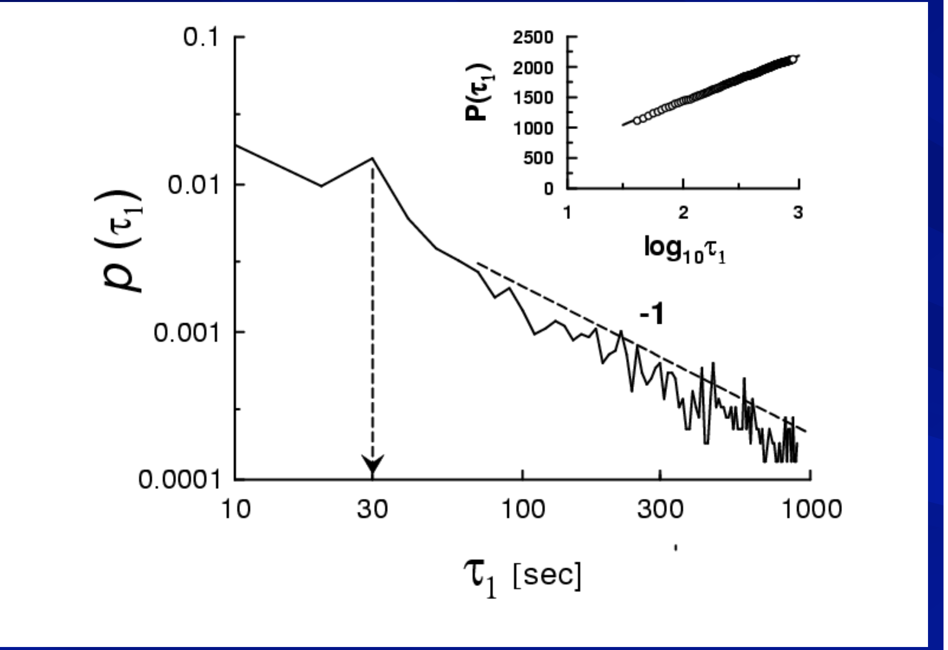
τ_1 = time between subsequent switches in the velocity signal

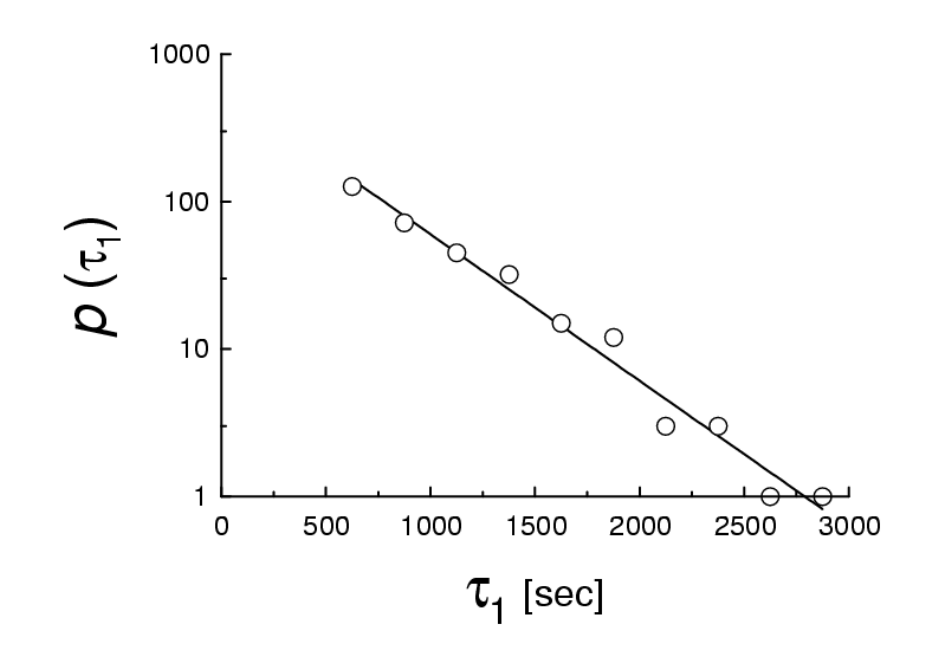
$$\tau_1 \equiv T_{n+1} - T_n$$

power-law scaling of the probability density function for small τ_1

for large τ_1: $p(\tau_1): \; \exp[-(\tau_1/\tau_m)]$

τ_m = 400 s

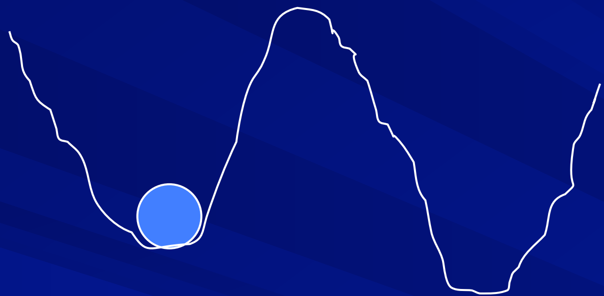




KRS, Bershadskii & Niemela, *Phys. Rev. E* **65**, 056306 (2002)

-1 power law scaling characteristic of SOC systems
(see papers in *Europhys. Lett., Physica A and PRE*)

Dynamical model



double-well potential

Balance between buoyancy and friction, forced by stochastic noise

For certain combinations of parameters, one obtains power-law for small times and exponential distribution for large times.

$$p(\tau_1) : \ \exp[-(\tau_1 / \tau_m)]$$

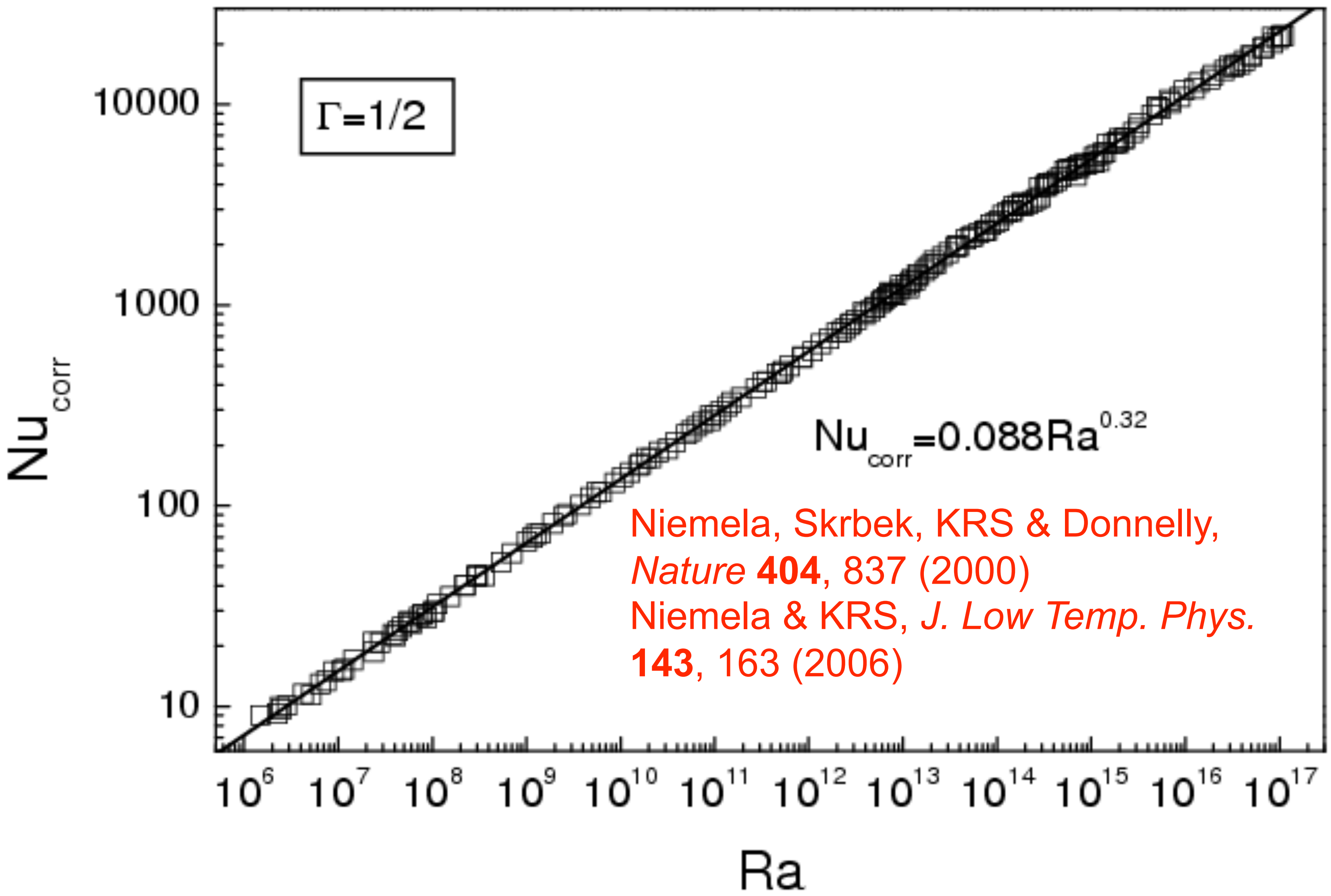
KRS, Bershadskii & Niemela, *Phys. Rev. E* **65**, 056306 (2002)

Summary of major points

- We have a fair number of definitive results about some model problems and know with empirical certainty about the real thing; much of the "classical" phenomenology appears to hold.
  - The classical predictions of the past have been confirmed (e.g., those relating to the -1 power).
  - The nature of anomalous scaling has been understood for the Kraichnan model, and may be true more generally.
- But there are gaps in our phenomenological understanding and questions remain. They can be posed sharply but have no sharp answers.
  - Why is the spectral constant for the Batchelor range twice as large as he determined?
  - What is the true effect of length scale ratio?

- Large scale features of the scalar depend on initial conditions quite severely, and each property has to be understood on its own merit. Models have been very helpful for understanding some essentials.
- Small scale scalar does not appear to be universal (more strikingly so than the velocity)
- Active scalars are illustrated through convection, where considerable progress is being made.

Thank you



Niemela, Skrbek, KRS & Donnelly, *Nature* **404**, 837 (2000)
Niemela & KRS, *J. Low Temp. Phys.* **143**, 163 (2006)

Upperbound results in the limit of $Ra \to \infty$

1. Arbitrary Prandtl number

- $Nu < aRa^{1/2}$ for all Pr (Constantin);
- a = 0.02634, according to Plasting & Kerswell, JFM 477, 363 (2003)
- Rules out, for example, $Pr^{1/2}$ and $Pr^{-1/4}$.

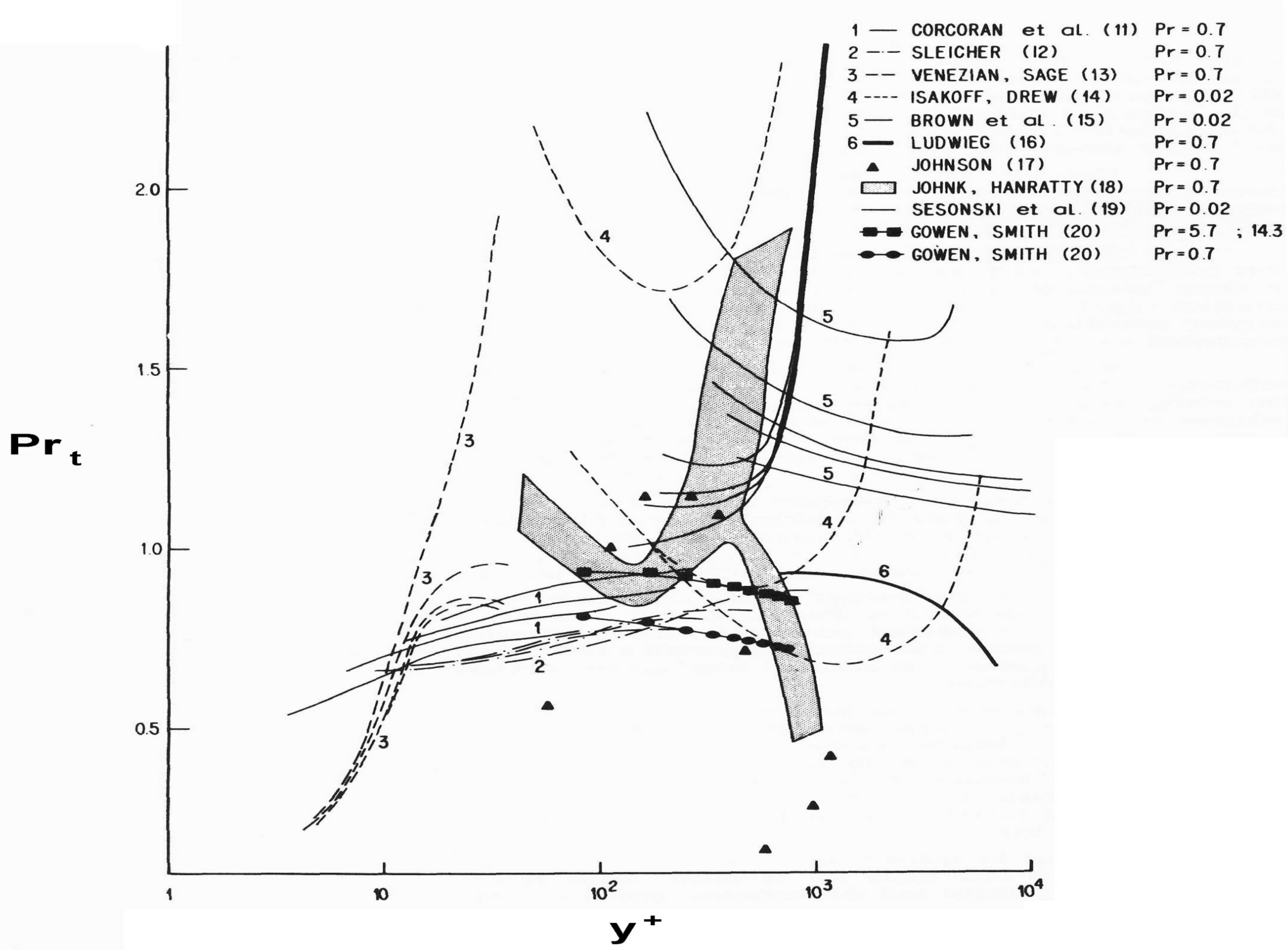
2. Large but finite Prandtl numbers

- For Pr > c Ra, $Nu < Ra^{1/3}(\ln Ra)^{2/3}$ (Wang)
- For higher Rayleigh numbers, the ½ power holds.

3. Infinite Prandtl number

- $Nu = BRa^{1/3}$ Howard, Malkus, mostly dimensional arguments, independent of the Prandtl number
- $Nu < CRa^{1/3}(\ln Ra)^{1/3}$ (Doering et al., exact)
- $Nu < aRa^{1/3}$ (Ierley, Kerswell & Plasting, JFM 560, 159 (2006)---"almost exact")

2 questions: Pr, 1/3 (2 views)



1 — CORCORAN et al. (11) Pr = 0.7
2 —·— SLEICHER (12) Pr = 0.7
3 — — VENEZIAN, SAGE (13) Pr = 0.7
4 ---- ISAKOFF, DREW (14) Pr = 0.02
5 — BROWN et al. (15) Pr = 0.02
6 — LUDWIEG (16) Pr = 0.7
JOHNSON (17) Pr = 0.7
JOHNK, HANRATTY (18) Pr = 0.7
SESONSKI et al. (19) Pr = 0.02
GOWEN, SMITH (20) Pr = 5.7 ; 14.3
GOWEN, SMITH (20) Pr = 0.7
Pr t
2.0
1.5
1.0
0.5
1
10
10²
10³
10⁴
y⁺