Progress in Pipe and Channel Flow Turbulence, 1961-2011

John Kim
Department of Mechanical & Aerospace Engineering
University of California, Los Angeles

TCM 2011, Marseille, France
September 26-30, 2011

Outline

- Prologue
- State of the Art - 1961
- Progress since TCM 1961
- Issues for Discussion
  - Mean velocity
  - Turbulence statistics
  - Turbulence structures
  - Effect of roughness
  - Some random questions
- Epilogue

Prologue

- A fully developed turbulent pipe or channel flow is, arguably, the simplest turbulent flow:
  - Stationary, exactly homogeneous in the streamwise direction
  - Worth noting that a fully developed turbulent channel flow is only approximately homogeneous in the streamwise direction, and it takes a large aspect ratio channel. An aspect ratio of 18, a la Hussain and Reynolds, requires 23 times larger flow rate.

- In a broader sense, almost everything we thought we knew about these simple wall-bounded turbulent flows has been a subject of further scrutiny.

- Many issues for pipe and channel flow turbulence also apply to turbulent boundary layers:
  - To what extent these canonical wall-bounded turbulent flows are similar in the wall region is of significant interest
  - Effects of the outer layer
  - Not homogeneous in the streamwise direction

The State of the Art – 1961

(presented by Hinze)

- Mostly on the mean velocity profile

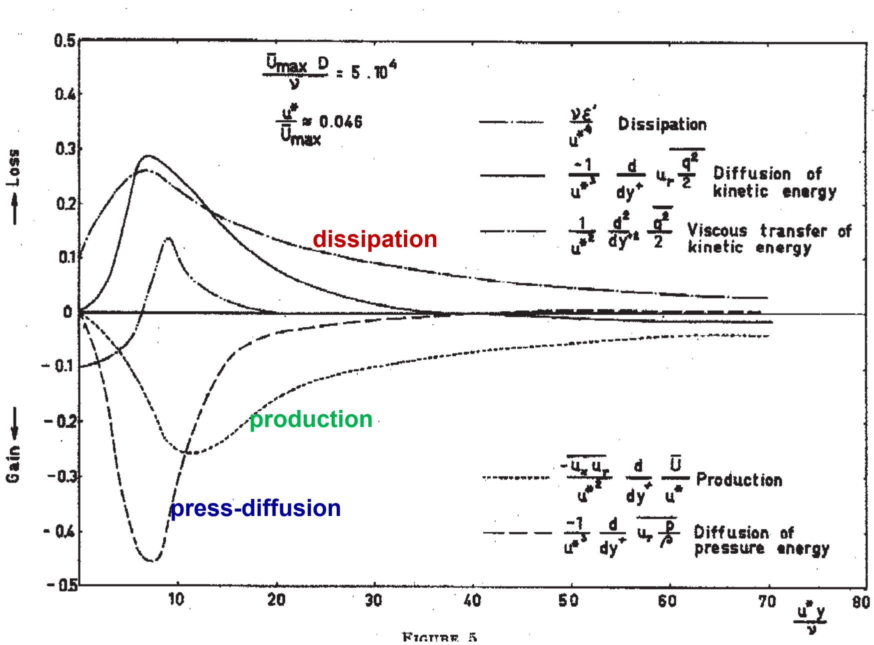
$$
\begin{aligned}
\frac{U}{u_\tau} &= A \log \frac{y u_\tau}{\nu} + B, \\
\frac{U_{\max} - U}{u_\tau} &= -A \log \frac{2y}{D} + B', \\
\frac{U_{\max} - U}{u_\tau} &= \phi\left(\frac{2r}{D}\right) = K \left(\frac{2r}{D}\right)^n
\end{aligned}
$$

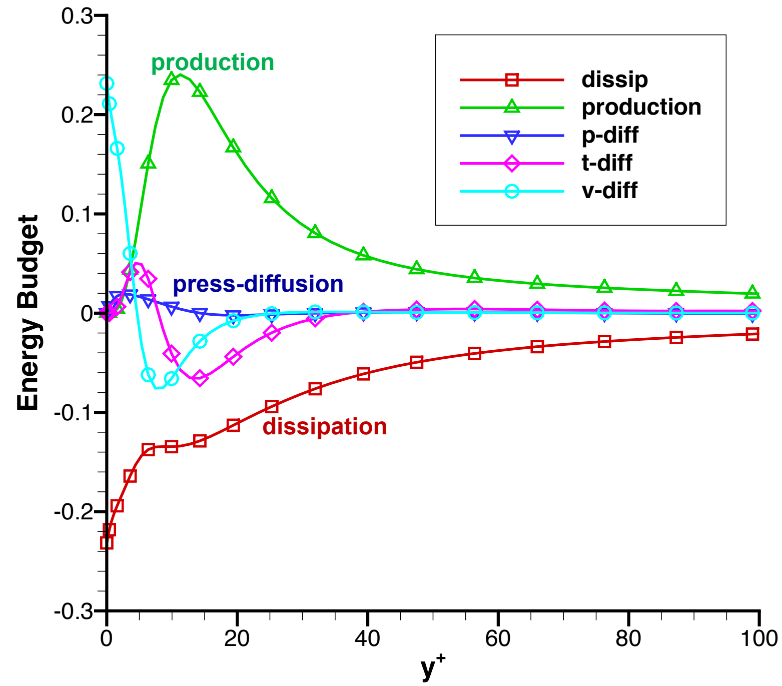
  - Observed a noticeable variation in the values of the constants, and raised the question of Re number dependence
  - "The law of the wall and the velocity-defect law might be only of approximate value, and the Von Karman constant might not be a constant and Reynolds number similarity might not exist or at the most at much higher Re numbers than assumed hitherto."
  - Examined the effects of roughness and concluded that single roughness parameter *k* is insufficient to describe the effect.
- Observed that the rms turbulence fluctuations normalized by u_τ are a function of Re

The State of the Art – 1961

(presented by Hinze)

- Energy balance in fully developed pipe flow (Laufer 1951)





Progress since TCM 1961

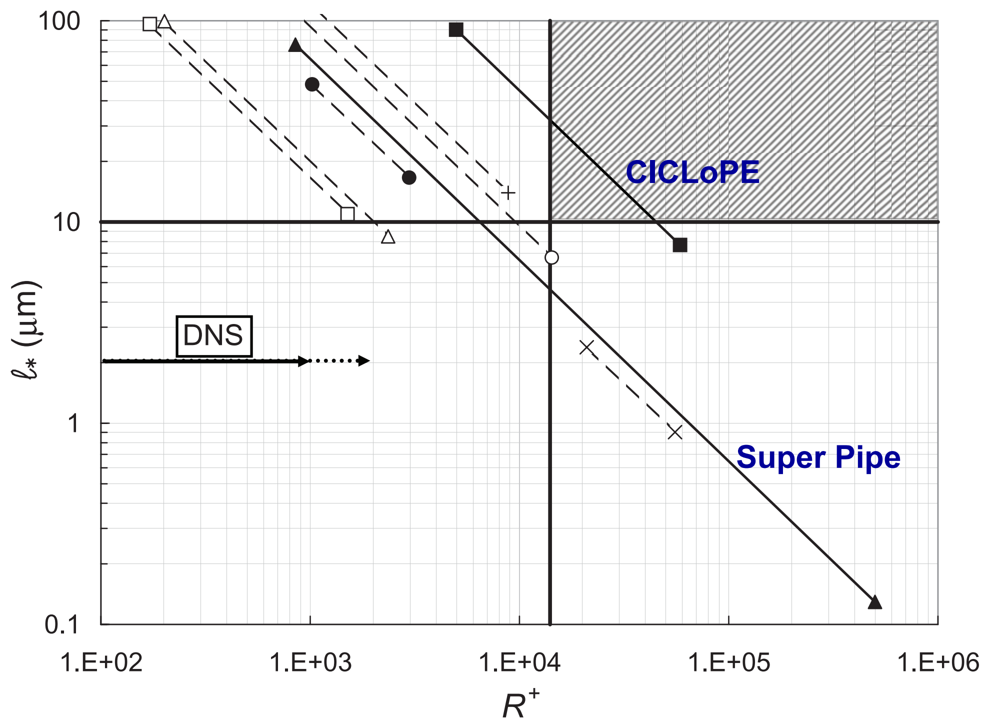
- **Experiments**
  - Comte-Bellot (1963): first extensive turbulence statistics, Re = 57000-230000
  - Hussain & Reynolds (1975): extremely long, high-aspect ratio channel, Re = 13800-33300
  - Eckelmann (1974): oil channel at low Re=2800-4100, turbulence structures near the wall
  - Johansson & Alfredsson (1982): detailed turbulence statistics, Re = 6900-24450
  - Zagarola & Smits (1998): Superpipe data, Re = 31×10^3-35×10^6
  - Zanoun, Durst & Nagib (2003): channel, up to $Re_\tau \approx 5000$ ($Re \approx 1.2 \times 10^5$)
  - McKeon et al. (2004, 2008): More Superpipe data and further analysis
  - Monty et al. (2009): a comparison of pipe, channel and boundary layer flows
  - Talamelli et al. (2009): Center for International Cooperation in Long Pipe Experiments (CICLoPE), 115 m long pipe with a diameter of 0.9 m, up to $Re \approx 3 \times 10^6$ ($Re_\tau \approx 60000$)

Design Considerations for CICLoPE

(0.9 m diameter, 115 m long pipe)

- The log region starts at $y^+ = 200$, and want to have at least a decade, up to $y^+ = 2000$.
- The log region extends up to 0.15 R^+, so R^+ should be at least 13300 (=2000/0.15)
- In order to have a sufficient range of Re over a factor of 3, up to R^+ =40000
- The smallest viscous length should be larger than 10 μm (hotwire with a diameter of 0.6 μm and the length 120 μm).

Fluid Dyn. Res. **41** (2009) 021407 A Talamelli *et al*



Progress since TCM 1961

- **Numerical simulations**
  - Deardorff (1970): large-eddy simulation (LES) of a turbulent channel flow, using only 6720 grid points
  - Schumann (1973, 1975): LES, using 65536 grid points
  - Both Deardorff and Schumann modeled the wall layer in their LES
  - Moin & Kim (1982): LES without modeled wall boundary conditions, using up to 516096 grid points for Re=13800 ($Re_\tau=640$)
  - Kim, Moin & Moser (1987): Direct numerical simulation (DNS) of a turbulent channel flow, using 3962880 grid points for Re=3300 ($Re_\tau=180$)
  - Eggels et al. (1994): DNS of a turbulent pipe flow, $Re_\tau=360$
  - Moser, Kim & Mansour (1999): DNS of turbulent channel flows up to $Re_\tau=590$
  - del Alamo & Jimenez (2004): DNS, $Re_\tau=934$
  - Hoyas & Jimenez (2006): DNS, $Re_\tau=2003$, using 1.8 x 10^{10} grid points
  - Wu & Moin (2008): DNS of pipe flows, up to Re=44000 ($Re_\tau=1142$) using 6.3 x 10^8 grid points
  - Some noteworthy numerical experiments
    - Jimenez & Moin (1991): The minimal flow unit with constrained spanwise scales
    - Jimenez & Pinelli (1999): The autonomous cycle without the outer layer
    - Kim & Lim (2000): The role of the dominant linear coupling term
    - Coleman, Kim & Moser (1995): True compressibility vs variable-property effect for high Mach number flows

Progress since TCM 1961

- **Theoretical Considerations**
  - Barenblatt (1993), Barenblatt & Chorin (1996): Scaling laws
  - Wosnik, Castillo & George (2000): Mesolayer and a log law for pipe and channel
  - Oberlack (2001): Lie group symmetry
  - del Alamo et al. (2004): Scaling of the energy spectra
  - George (2007): A universal log law for turbulent wall-bounded flows?
  - Panton (2007): Composite asymptotic expansions
  - McKeon & Morrison (2007): Asymptotic scaling
  - Nagib & Chauhan (2008): Analysis using a composite mean velocity profile

Mean Velocity Profile

- This (or a similar variation) is what we normally teach:
  - In the inner region,
$$U = f(y, u_\tau, \nu)$$
  - In the outer region,
$$U_{\max} - U = g(y, R, u_\tau)$$
  - Dimensional analysis leads to,
$$U^+ = f(y^+), \quad (U_{\max} - U)/u_\tau = g(y/R)$$
  - For large Re, there would be a region, $\nu/u_\tau << y << R$,
$$U^+ = A \log y^+ + B$$
  - The log profile is universal, i.e., the two constants are independent of Re.

- Barenblatt (1993):
  - Incomplete similarity for large but finite Re leads to a power law,
$$U^+ = C\,(y^+)^\alpha = \left(\frac{1}{\sqrt{3}} \ln Re + \frac{5}{2}\right)(y^+)^{\frac{3}{2 \ln Re}}$$
  - Constants depend on the flow Re, indicating that its dependence on viscosity due to incomplete similarity

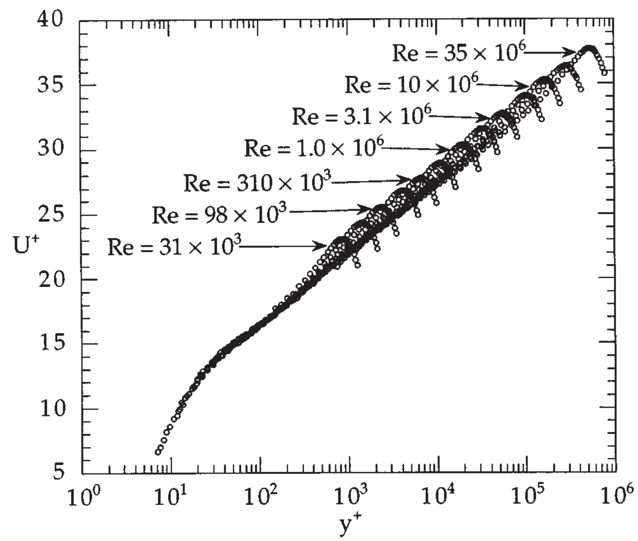
Recapitulating the power-law vs log-law arguments

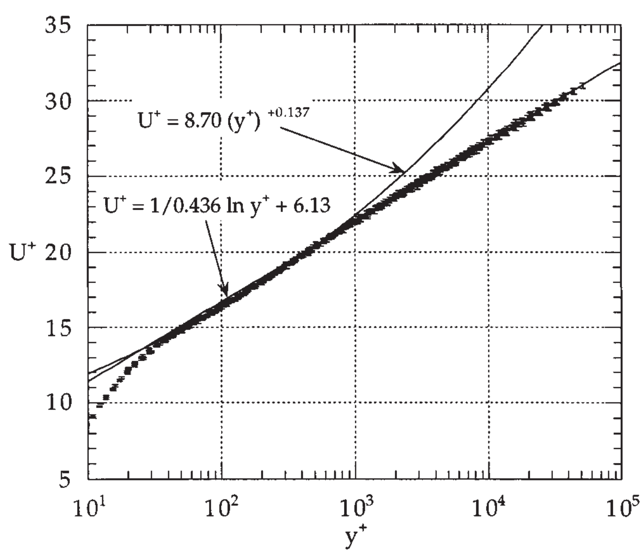
- Power law (Barenblatt):
  - Both a power law and a log law have equally rigorous theoretical foundations, i.e., not convenient representations of empirical data.
  - Due to incomplete similarity for large but finite Re, the dependence of the velocity gradient on the molecular viscosity does not disappear, and hence a power law
  - A power applies to $40\,\nu/u_\tau \le y \le 0.9\,R$
  - A universal log law is related to the envelope of a family of power-law curves, each corresponding to a fixed Re
  - Used, among others, the Princeton Superpipe data to advocate a power law
- Log law (Zaragola & Smits):
  - Using the same data they concluded that a power law and a log law coexisted
    - For low Re, $R^+ \le 5000,$ a power law in $50 \le y^+ \le 0.1\,R^+$, followed by a log law
    - For high Re, a power law in $50 \le y^+ \le 500$, followed by a log law in $500 \le y^+ \le 0.1\,R^+$
- Barenblatt contended that the Superpipe data for high Re are affected by surface roughness
- Smits & Zagarola confirmed that the effect of roughness on their data is negligible
- Wosnik et al. (2000):

$$U^+ = \frac{1}{\kappa\,(R^+)} \ln\left[y^+ + a^+\left(R^+\right)\right] + B(R^+)$$

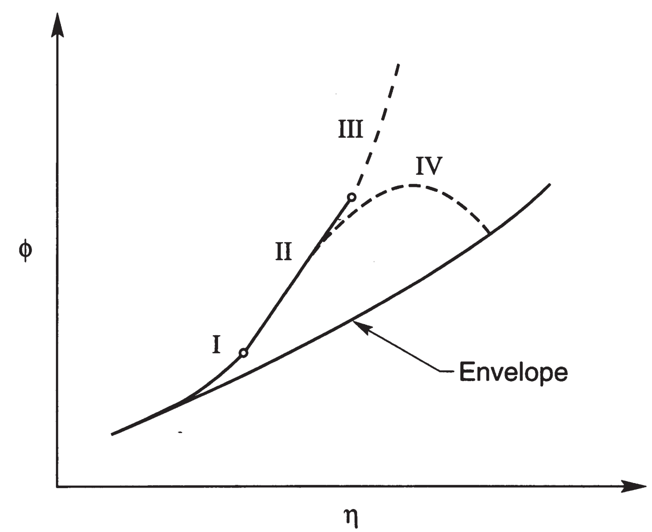
Mean Velocity Profile

Power law vs log law





Zagarola et al. (1977), up to y less than 0.1 R



Barenblatt & Chorin (1996)

Mean Velocity Profile

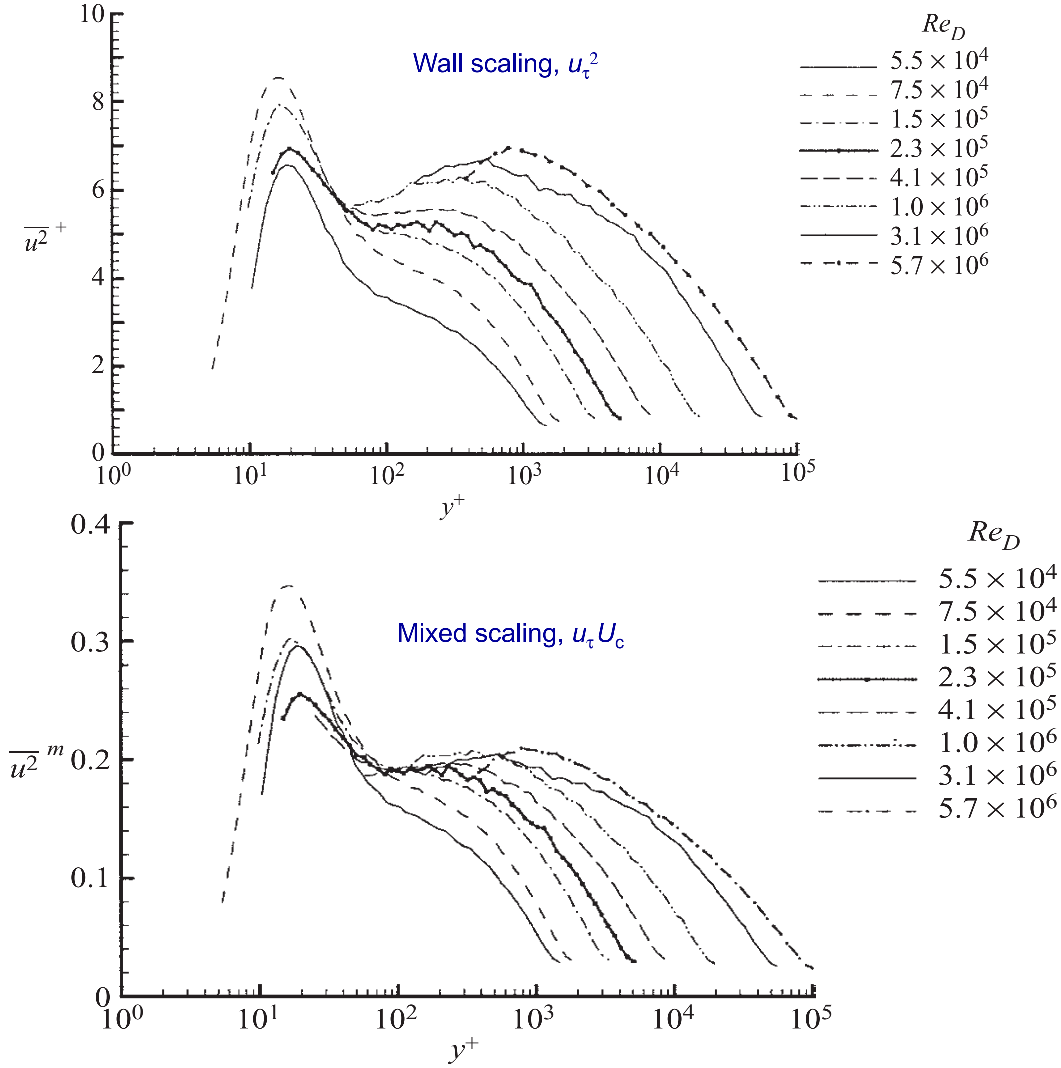
- Is there a log law? If so, is it universal? Are they flow dependent?
- What is the minimum Reynolds number above which the log region exists unambiguously?
  - Where does the log region start and end? A general rule of thumb is $100\,\nu/u_\tau \le y \le 0.15\,R$
    - Does it depend on Re?
    - Does it depend on the BC (ie, BL vs pipe or channel)?
    - Based on the mean velocity only? How about, for example, $\ell \sim y$?
- Is the power law with Re-dependent coefficients a better scaling law for high but finite Reynolds numbers?
- What is the proper velocity scale for the defect law for the mean velocity in the outer region?
  - The previously accepted velocity scale, the wall-shear velocity, u_τ, has been challenged by many investigators, who have proposed a mixed velocity scale or a velocity scale based on outer velocity, such as U_c or U_c -U_m.
  - Does $U_c - U_m \rightarrow \text{const} \times u_\tau$ in the limit $\mathrm{Re} \rightarrow \infty$, thus leading to the same velocity scale asymptotically?
- If the proper velocity scale for the outer layer is other than the wall-shear velocity, is a log law still possible?
  - Note that the usual simple matching argument would not lead to a log law unless the defect law is also scaled by the wall-shear velocity, but a log law is still plausible as the leading term in a logarithmic expansion (Wosnik etal. 2000).
- Is the wall region, including the viscous and inertial layer in fully developed pipe flows, inherently different from that of turbulent boundary layers over a flat plate (i.e., zero pressure gradient boundary layer)?
  - If so, is the difference due to different outer layers?
  - Or is it due to homogeneity vs inhomogeneity?
  - Or is it simply due to the presence of the overall force balance in fully-developed internal flows (i.e., pressure gradient is balanced by the wall shear), thus rendering the wall-shear velocity to be the proper velocity scale for both inner and outer regions?

Turbulence Statistics

- What is the proper velocity scale for turbulence statistics in the wall region?
  - The peak rms value of streamwise-velocity fluctuations when scaled by the wall-shear velocity, u_τ , increases with the Reynolds number, thus suggesting an effect of outer-scale motions.
  - What is the peak rms value in the limit $\mathrm{Re} \to \infty$?
  - Is the failure of the inner-layer scaling (i.e., scaling by u_τ) due to Re-dependent attached eddies (*a la* Townsend)?
  - Although the peak rms value scaled by u_τ increases with Re, the location of the peak appears to be independent of Re, $y^+ \approx 15$, which is also the location where the production peaks independent of Re. Why?
  - Is the second peak of the streamwise-velocity fluctuations real? Or should it be a shoulder (instead of a peak), and the peaks observed in some experiments are actually due to a probe-resolution problem?
  - The second peak appears only at high Reynolds numbers. Does this suggest a significant change in turbulence characteristics beyond a certain Reynolds number?
  - Should streamwise and spanwise components be scaled differently from the wall-normal component? Is this simply due to the fact that the wall affects the wall-normal component differently?
  - The location of the maximum Reynolds shear stress normalized by the inner variables increases with Re, but the location of the maximum production seems to be independent of Re, $y^+ \approx 15$. Why?
  - Should Reynolds stresses in the wall region in internal flows be different from those in turbulent boundary layers, since they are influenced by different LSMs and VLSMs in the outer layer?
  - The fraction of TKE in the wall region, where the first peak is located, decreases as Re increases. Does this mean that the wall region becomes less important at high Reynolds numbers?

Streamwise velocity fluctuations

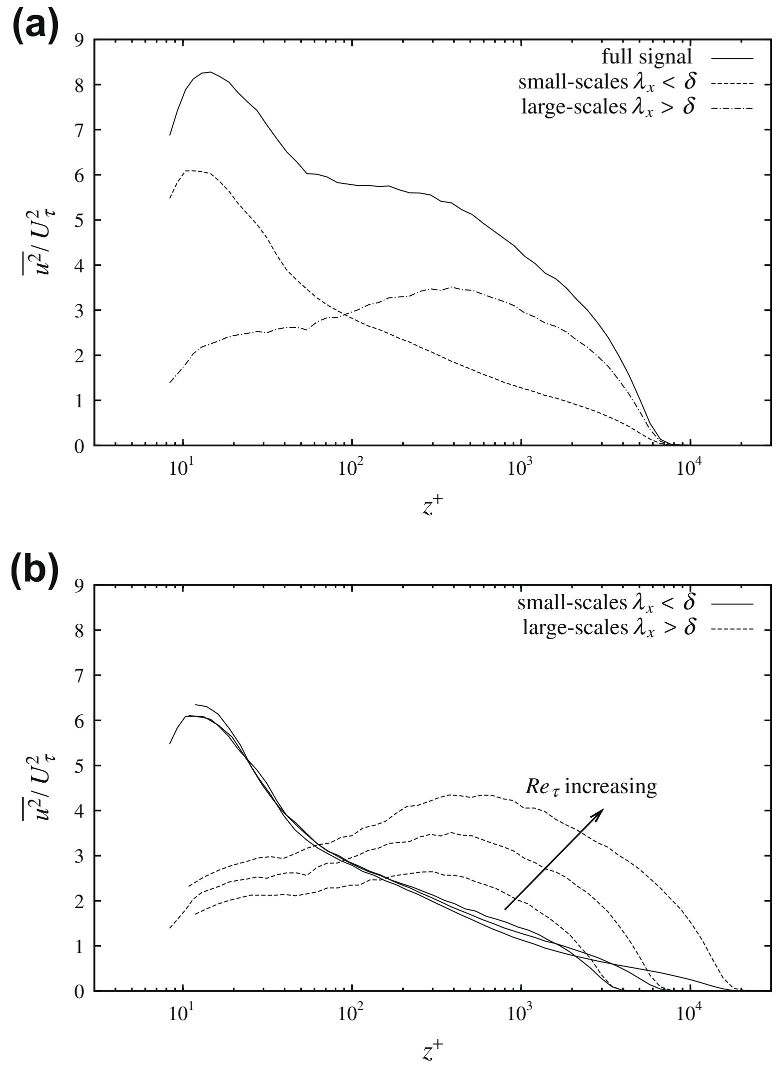
Morrison et al. (2004)



Streamwise velocity fluctuations

Marusic et al. (2010) – **note these are TBL data**

- Fluctuations are decomposed into two parts
  - Small scales, universal in wall scaling
  - Large scales, Re dependent

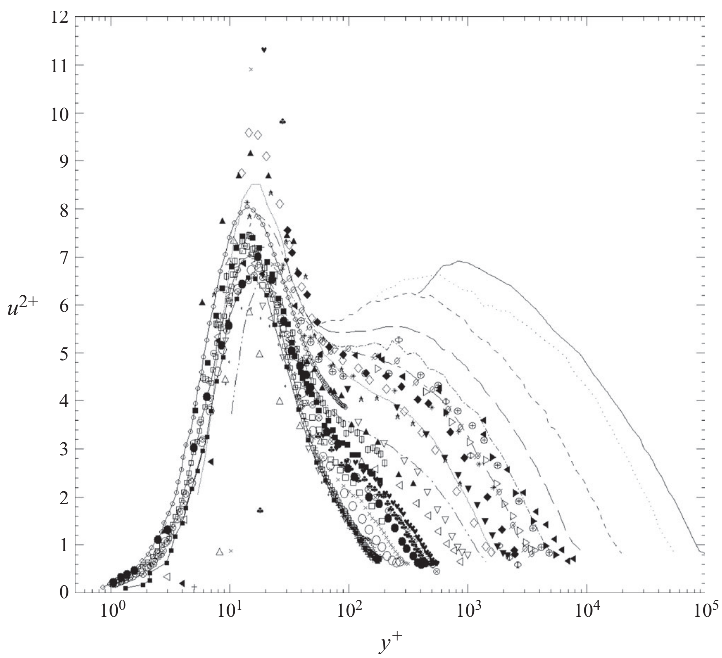


Streamwise velocity fluctuations

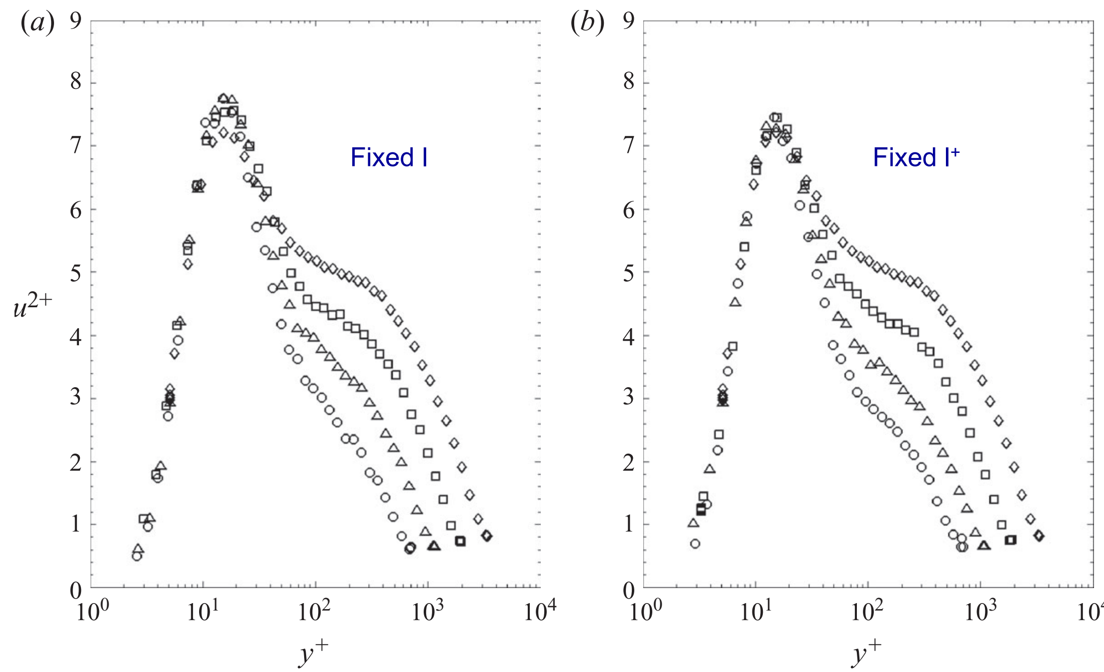
Hultmark, Bailey & Smits (2010)

- No Re dependence of u' scaled by u_τ for pipe and channel flows?
  - If true, no influence by LSMs or VLSMs in contrast to TBL?

Various sources



Hultmark et al.



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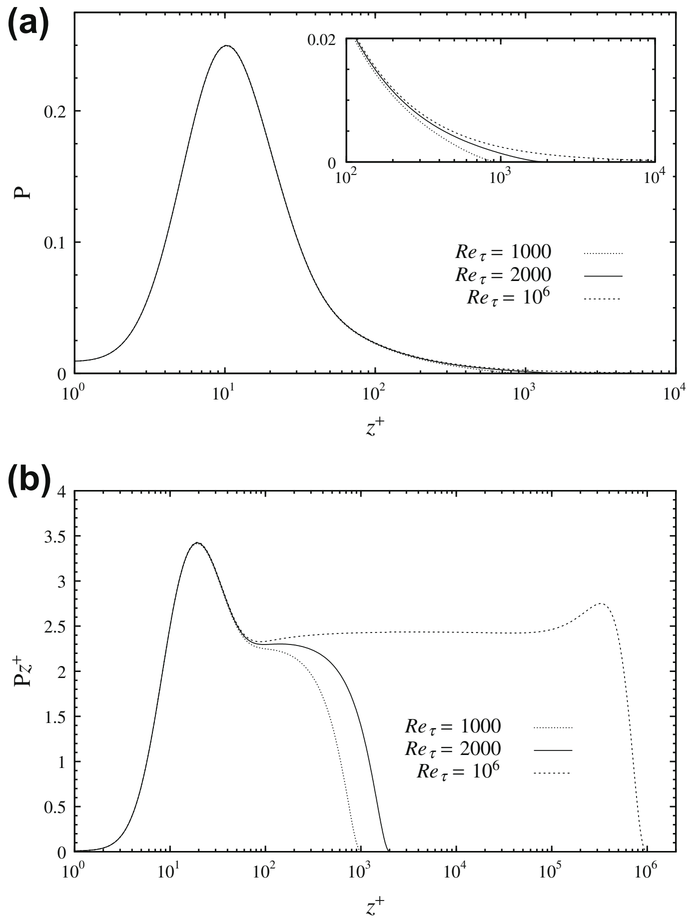
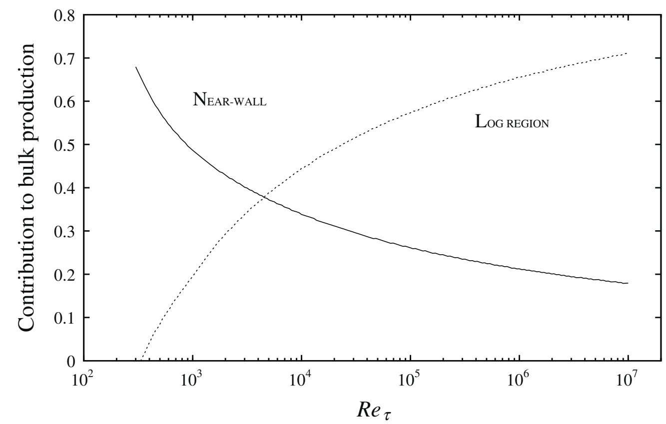
TKE Production

Marusic et al. (2010)





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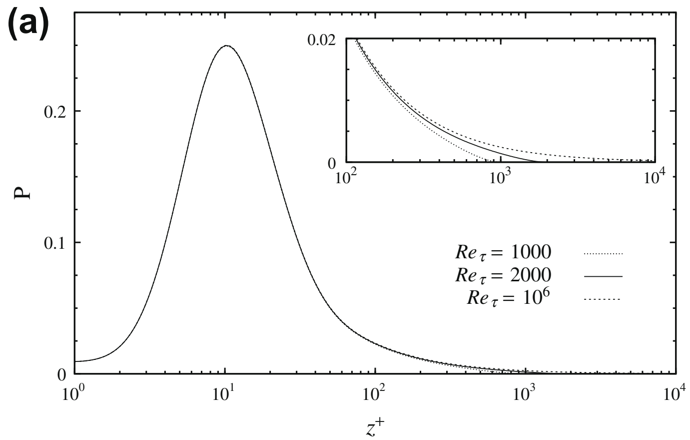
Marusic et al. (2010)



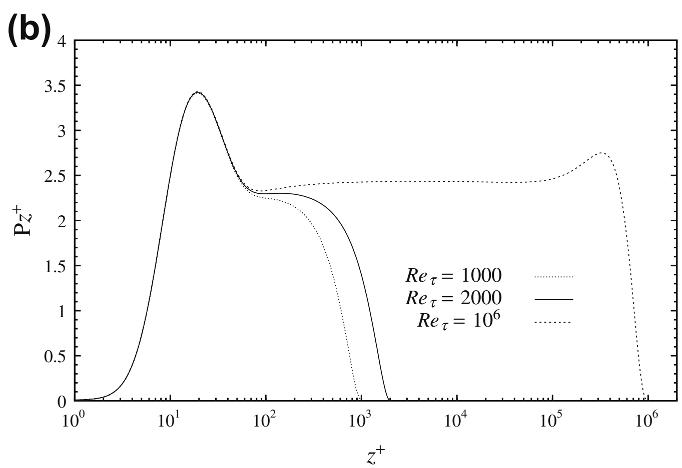
For fully developed channel or pipe flow

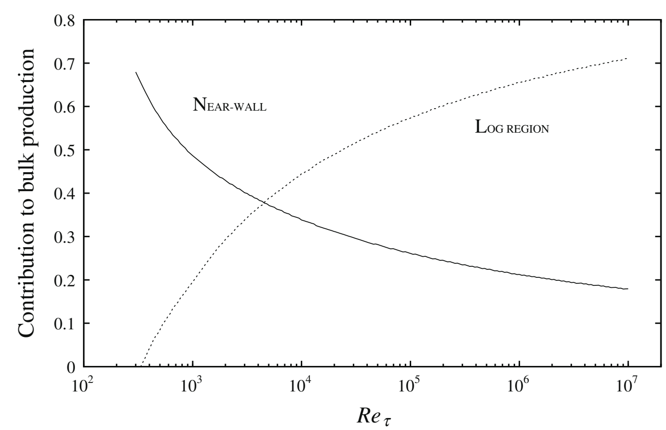
$$
\begin{aligned}
-\overline{uv} + \frac{dU^+}{dy^+} &= 1 + \frac{1}{Re_\tau} y^+ \\
&\approx 1
\end{aligned}
$$

TKE production can be written as

$$P^+ = -\overline{uv}\frac{dU^+}{dy^+} = \frac{dU^+}{dy^+}\left(1 - \frac{dU^+}{dy^+}\right)$$

Thus, P^+ has its maximum 0.25 where $dU^+/dy^+ = 0.5$.





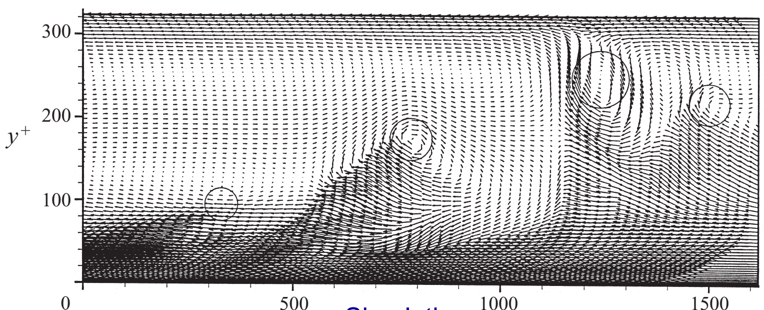
Turbulence Structures

- Are turbulence structures in the wall region (e.g., streaks and streamwise vortices) universal independent of the outer flow?
  - In spite of the recent observation of the organized large-scale motions and their strong influence on near-wall turbulence, spanwise spacing of the wall-layer streaks normalized by wall variables appears to be independent of Re and flows. Is this true, and why?
- Recent observations suggest that LSMs and VLSMs interact (or modulate) with near-wall turbulence. This seems to contradict the classical notion (*a la* Townsend) that these large-scale outer motions are "inactive" motions, i.e., they do not contribute to the Reynolds shear stress.
- It has been shown that LSMs and VLSMs are different for internal and external flows. If these different structures influence (or modulate) turbulence across the whole width (or boundary layer), different wall-layer dynamics are expected, but they appear to be very similar to each other. Why?
- Are VLSMs real, or an experimental or numerical artifact?
  - It is worth mentioning that the detection of VLSMs are mostly based on Taylor's hypothesis, the applicability of which (to large scale motions) is subject to a critical examination.
- Is the self-sustaining mechanism of near-wall turbulence universal, independent of Re and the outer flow?
- Some investigators contend that the role of LSMs and VLSMs would become more significant as Re increases, to the extent that the bottom-up process prevalent at low Re would be replaced by a top-down process. What evidence do we have to support this notion?
  - What are the implications for flow control at high Reynolds numbers?
  - How do we reconcile the success flight test in which riblet surface, which was designed to control near-wall structures, achieved drag reduction at high Re?

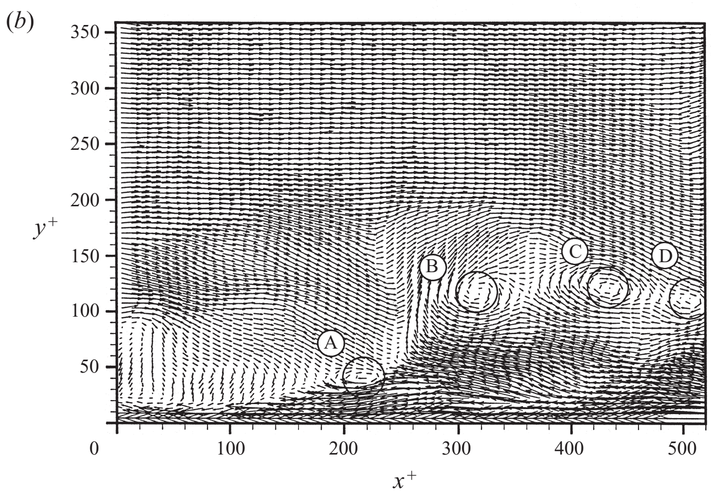
Large Scale Motions (LSM)

Zhou et al. (1999)

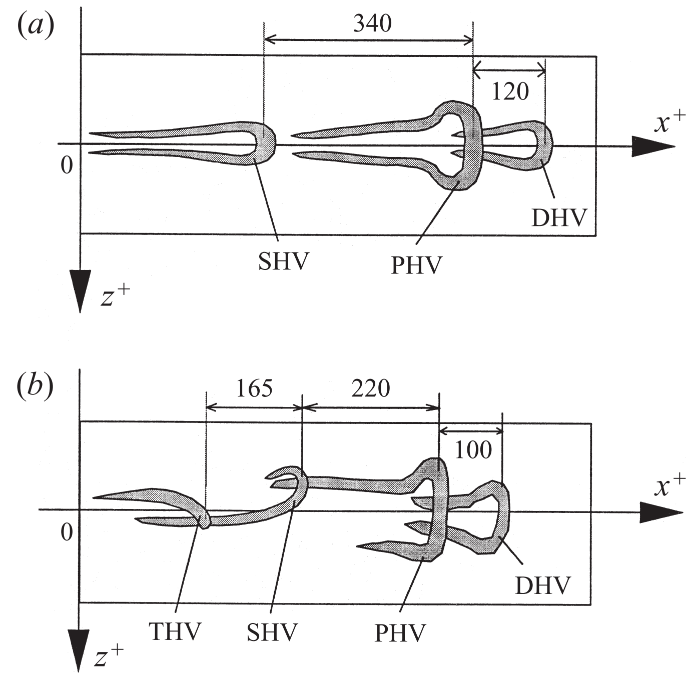
- Discovery and characterizations of coherent structures (e.g., streaks, streamwise vortices, horseshoe/hairpin vortices, etc.) were a major milestone in 70s and 80s.
- Discovery of large-scale motions (LSM) has received much attention since 90s
  - LSM is formed when hairpin vortices align coherently in packets that are about 2*R* long



Simulation



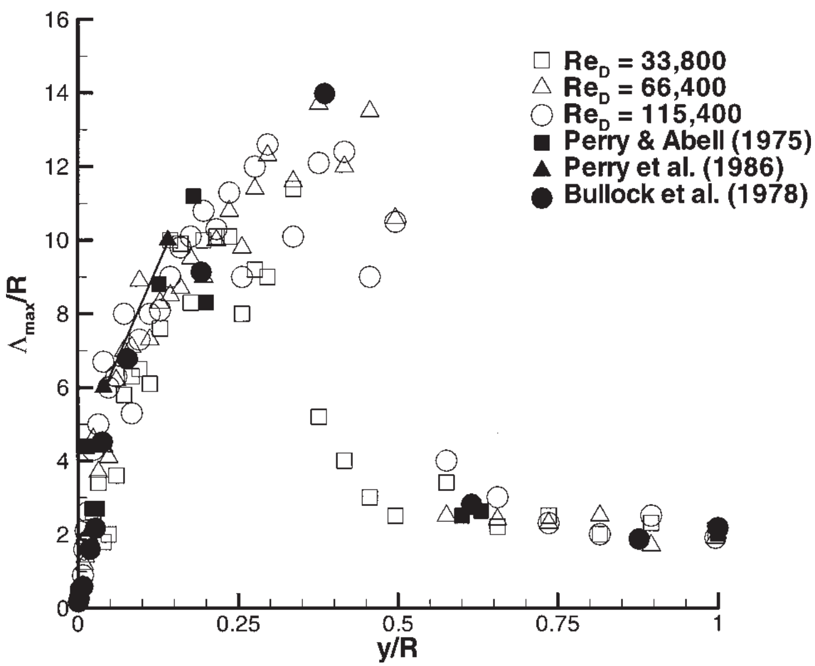
Experiment

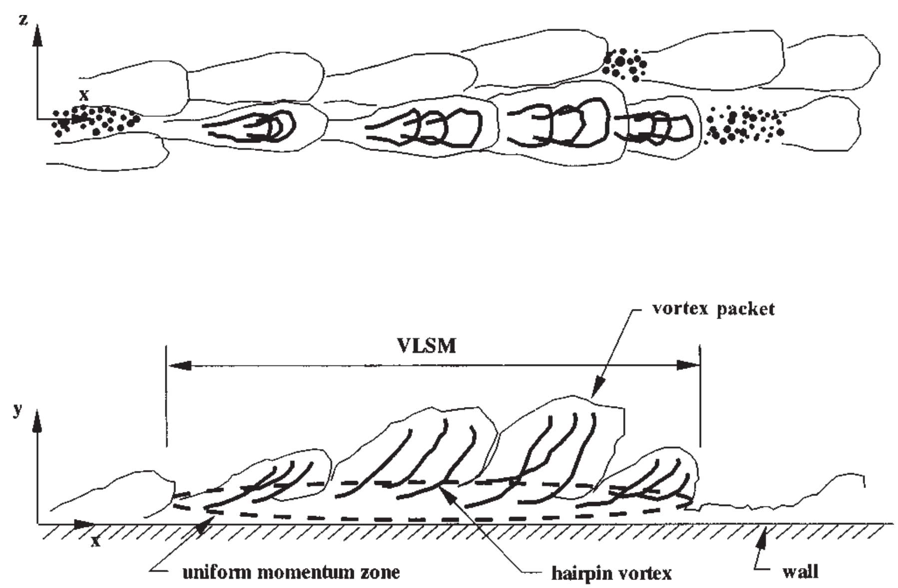


Very Large Scale Motions (VLSM)

Kim & Adrian (1999)

- VLSM is a consequence of large-scale motions associated with packets of hairpin vortices aligning coherently so that the low momentum flow from the lower half of one is passed on to the next, and so on over a span of many LSMs





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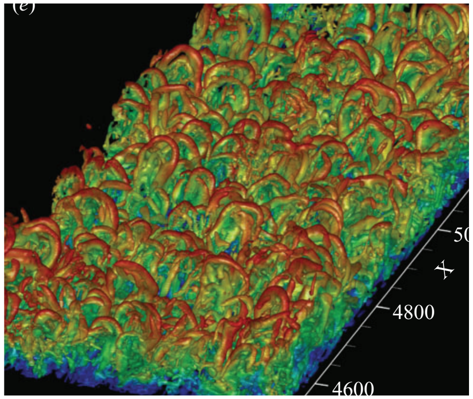

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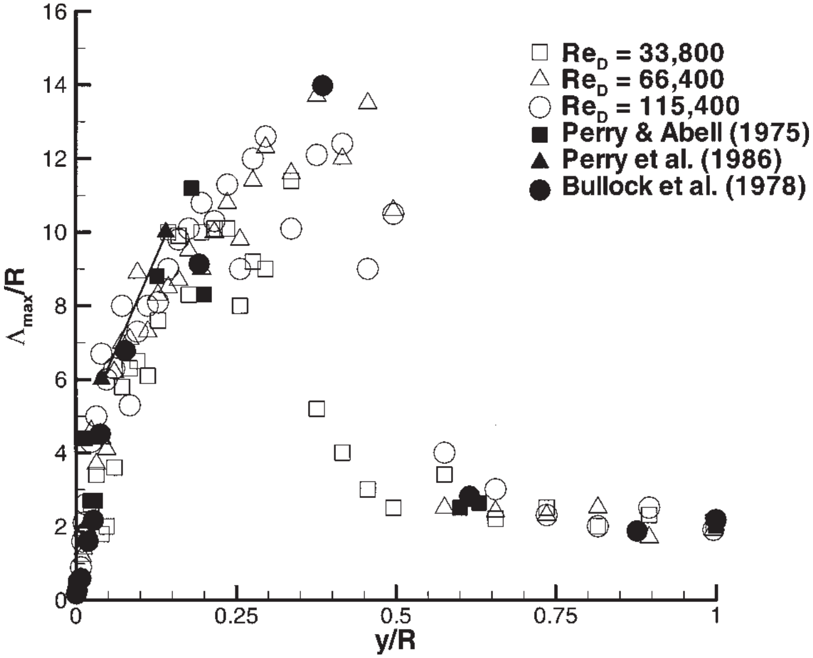
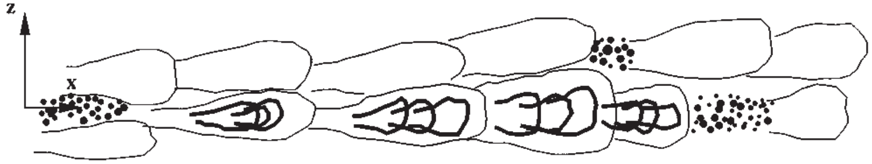
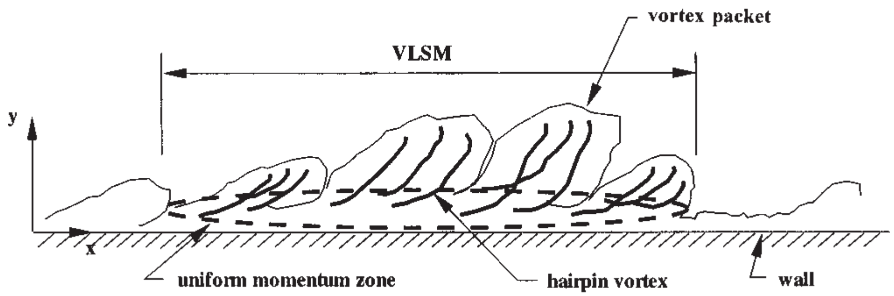


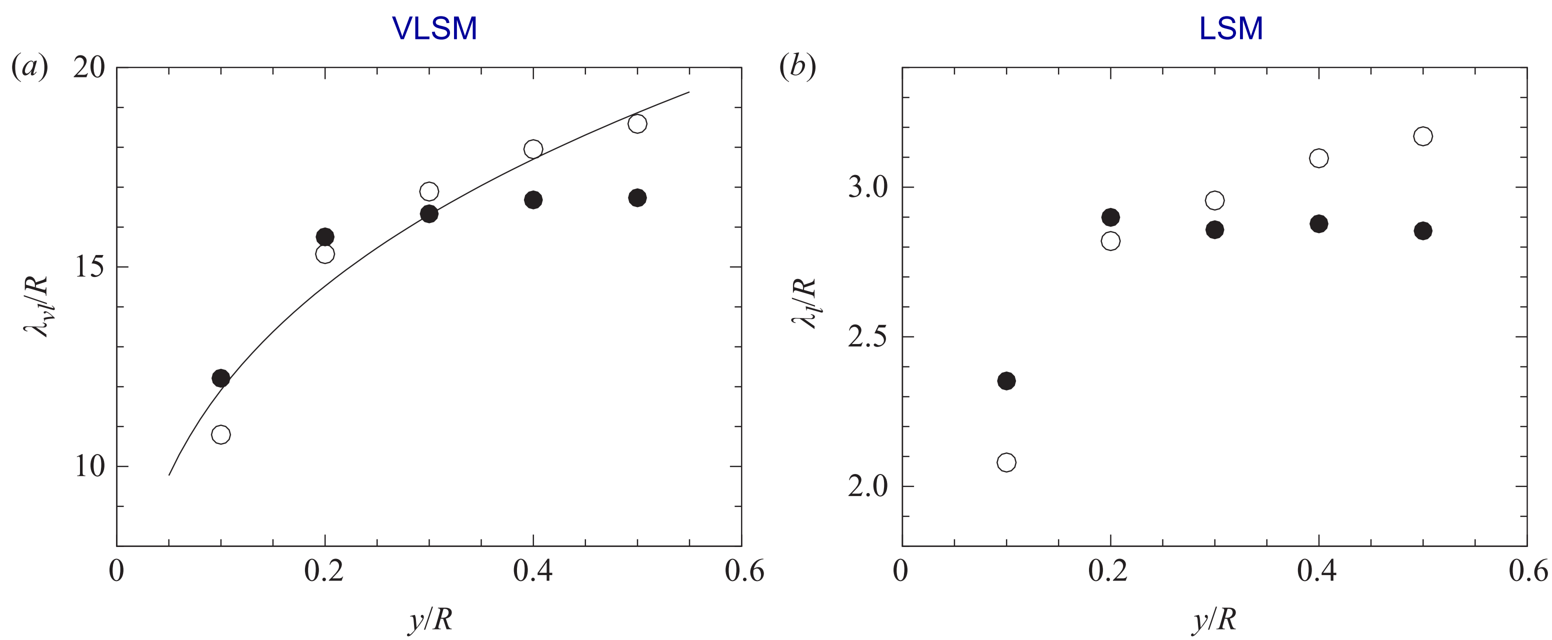




Very Large Scale Motions (VLSM)

Bailey & Smits (201)

- Detached LSM in the outer layer
- Attached LSM near the wall
- If the VLSMs are caused by the alignment of the LSMs, only the LSMs in the outer layer are involved
- The VLSMs have large radial scales which result in a strong correlation with motions near the wall, supporting the notion of the modulation of the near-wall flow forwarded by Hutchins & Marusic (2007) and Mathis et al (2009)



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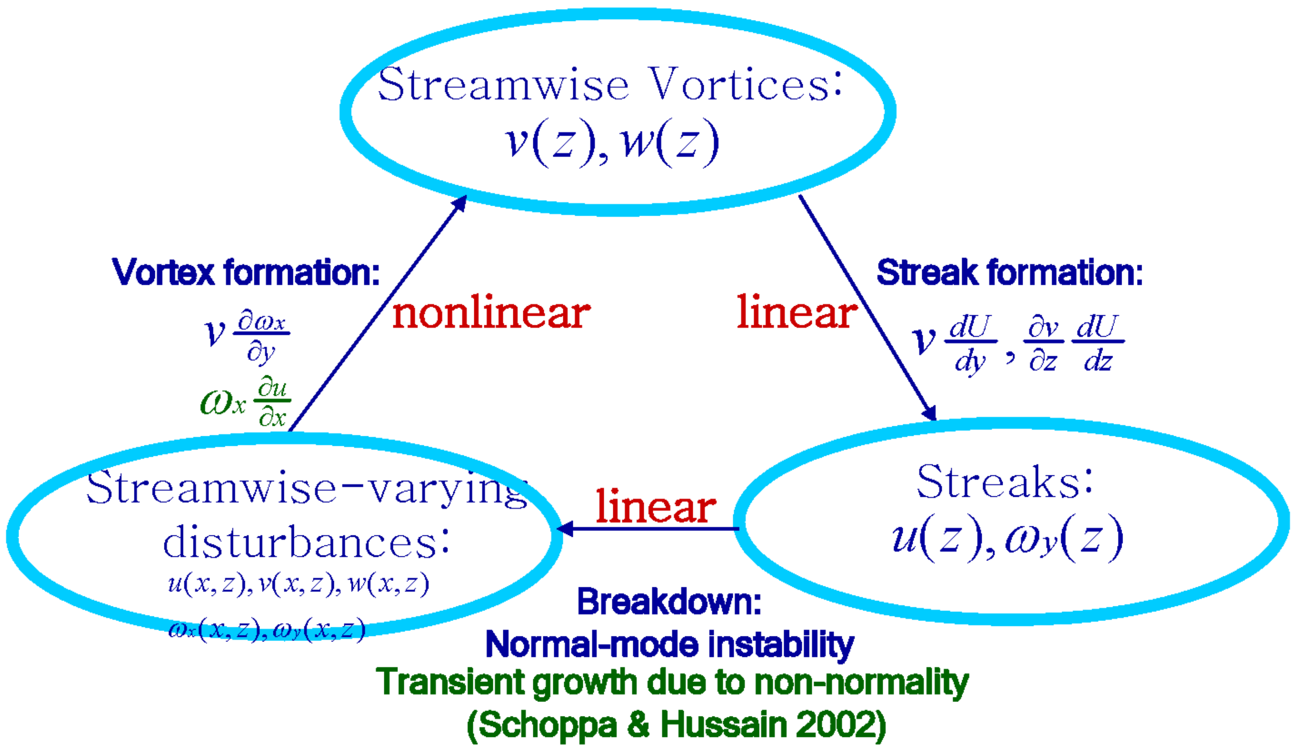
Self-Sustaining Mechanism

Hamilton et al. (1995)



Effects of Roughness

- Understanding the effects of surface roughness has attracted more attention recently as our interest for high Re flows grows.
- The commonly used Colebrook's formula (via the Moody chart, for example) predicts a monotonic effect of surface roughness on the drag in transitional to turbulent regime. However, new experimental data show a different trend in the transitional regime, which is consistent with Nikuradse's data.
- Is the notion of a hydrodynamically smooth surface for, say $k_s^+ \leq 4$, valid?
- Some data suggest that the effects of surface roughness in internal flows are different from those in external flows. If so, why?
- The classical notion (aka Townsend's hypothesis) that the influence of surface roughness is confined to the wall region and that the outer flow is independent of surface roughness seems to have survived recent debates.
- There have been some anomalies, but it is now generally accepted, perhaps except for two-dimensional roughness, which seem to affect the flow further beyond the wall region compared to three-dimensional roughness, that the outer flow is unaffected by surface roughness provided that the roughness height (usually measured in terms of the equivalent sand-grain roughness)I s small compared to the pipe diameter (or channel height, or boundary layer thickness for external flows).
- The above conclusion is based primarily on the mean velocity profile and the rms turbulence fluctuations. It has not yet been demonstrated whether the same holds for higher-order statistics.
- How far does the roughness-affected layer extend?
- If Townsend's hypothesis holds for turbulence structures as well, how do we reconcile the recent recognition that interactions between outer flow large-scale motions and inner flow turbulence are much stronger than previously understood?
  - Does this mean that the interactions are one-way, i.e., only the outer flow can influence the inner flow, but not the other way around?

Transitional Roughness

- Mean velocity profile for rough wall,

$$U^+(y) = \frac{1}{\kappa}\log y^+ + B - \Delta U^+ + \frac{\Pi}{\kappa}w$$

- Hama function, ΔU^+, is a function of k_s^+ (actually the equivalent sand-grain roughness is defined in this way)
- Hydrodynamically smooth surface?

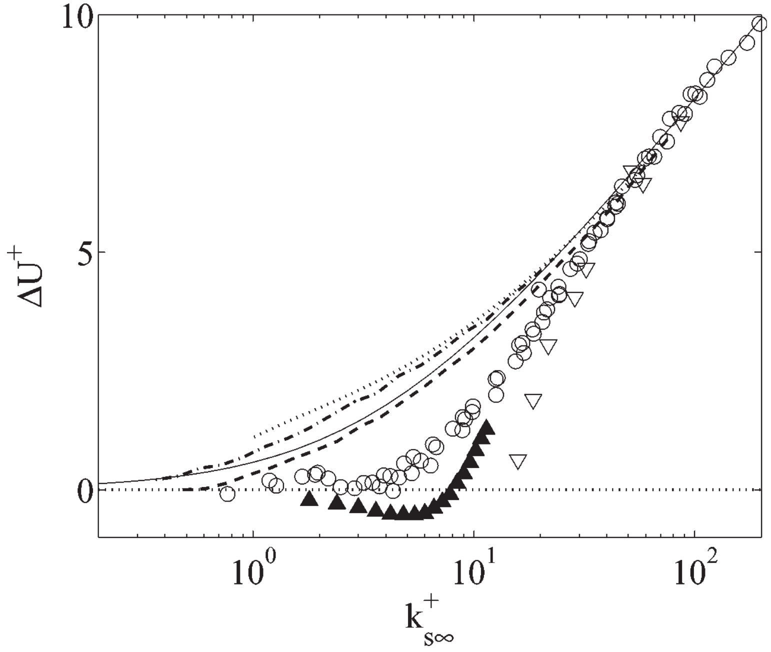
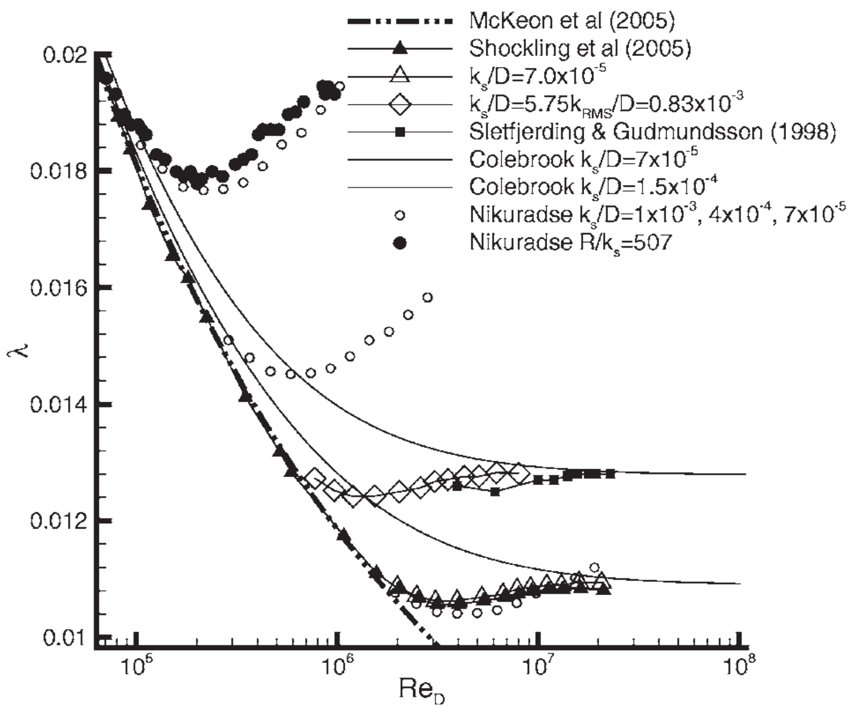


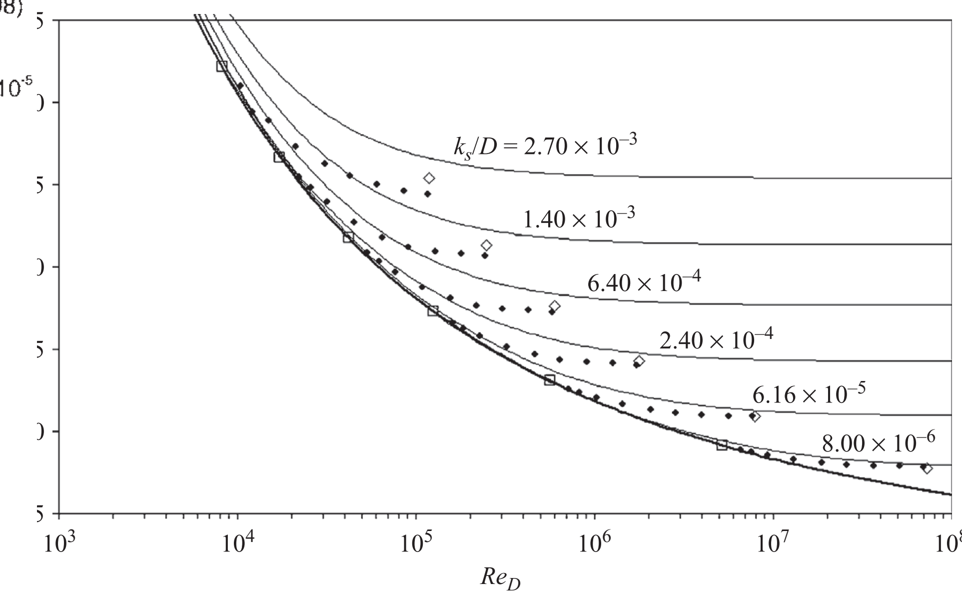
Figure 3 Roughness function for several transitionally rough surfaces, as a function of the Reynolds number based on the fully rough equivalent sand roughness. ○, uniform sand (Nikuradse 1933); ▽, uniform packed spheres (Ligrani & Moffat 1986); ▲, triangular riblets (Bechert et al. 1997); ······, galvanized iron; – – – –, tar-coated cast iron; —·—, wrought-iron (Colebrook 1939); ——, Equation 12.

Friction Factor for Rough Surfaces

- In contrast to the behavior seen in the Moody plot, the friction factor in the transitionally rough regime follows an inflectional sand-grain-roughness-type (Nikuradse like) distribution rather than the monotonic Colebrook relationship.



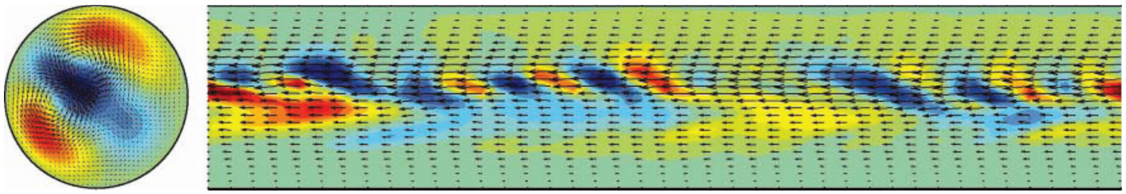
Honed surfaces and most roughness types



Commercial steel pipes

Some Random Questions

- What is the required probe resolution for turbulence measurements?
  - Is the generally accepted value, $\ell^+ < 10$, sufficient?
  - In a similar vein, what is the required spatial (and temporal) resolution for numerical simulations?
    - Is the numerical grid of several wall units, which is typical for most direct numerical simulations, sufficient?
    - The usual justification that most dissipations should be captured may not be sufficient in addressing certain subtle issues raised above (e.g., the second peak in the rms fluctuations), in part because this justification is based on the average dissipation (i.e., Kolmogorov scale) while much smaller scales may present locally and instantaneously.
- Is there any link between LSMs/VLSMs and the exact but unstable traveling wave solutions, aka ``exact coherent structures'' (Eckhardt et al. 2007)?
  - Although these exact traveling wave solutions have been observed only at low Reynolds number flows, there is no reason why these solutions cease to exist at higher Reynolds numbers.



- Fully developed pipe flows and channel flows are generally considered to be the same for all purposes, and yet their linear stability is the opposite, the former being linearly stable for all Reynolds numbers while the latter has a critical Reynolds number beyond which the flow is linearly unstable. Are there any differences in fully developed turbulent flows as a consequence of different linear stability?

Epilogue

- Much progress has been made over the past half century, yet turbulence has been elusive refusing to reveal its true nature.

- In spite of reliable experimental data at high Re and detailed numerical data at low to moderate Re, many fundamental issues still remain to be resolved.

- It will take a large-scale collaborative experimental effort in order to address unresolved issues. The CICLoPE is a right step toward this effort.

- In spite of the remarkable advancement of modern supercomputers, the required computing power for simulation of turbulent flows at high Reynolds numbers is much beyond that of presently available computers (and those feasible in the near future).

- It would take a concerted effort by the whole community (theoreticians, experimentalists, computational fluid dynamicists) to improve our understanding of the true nature of turbulent flows, and to enhance our ability to manipulate them into a state with desired properties.

Epilogue: TCM 2061?

- Someone said that **prediction about the future is a fool's exercise**, but so that they can have a good laugh at Turbulence Colloquium Marseille 2061
- Topics, related to pipes/channels and BLs only, to be discussed in TCM 2061 may include
  - How high Re is high enough?
  - Highest Re simulation would be $Re=10^{?}$?
  - A new unified law instead of a log or power law
  - EVLSM, and a pipe will never be long enough and a computational domain will never be large enough
  - Numerical simulations and LDV/PIV have changed the way we study turbulent flows since TCM 1961. Whatever the next major breakthrough may be, they will be analyzing turbulence with the next generation technology (3D TPIV, 3D holography virtual wind tunnel, high-level language to solve the N-S equations, etc.)
  - Still trying to achieve turbulent pipe flows with a laminar-like drag
  - And so forth …

Epilogue: TCM 2061?

- But by 2061 we all would be in heaven looking down those guys at TCM 2061, and we will also have a good laugh.
- And we would have already figured out turbulence by talking to God.
- On the other hand, like Horace Lamb once said, I am not too optimistic about it.
- Finally, you can ask questions, and I hope someone in the audience may have the answer.

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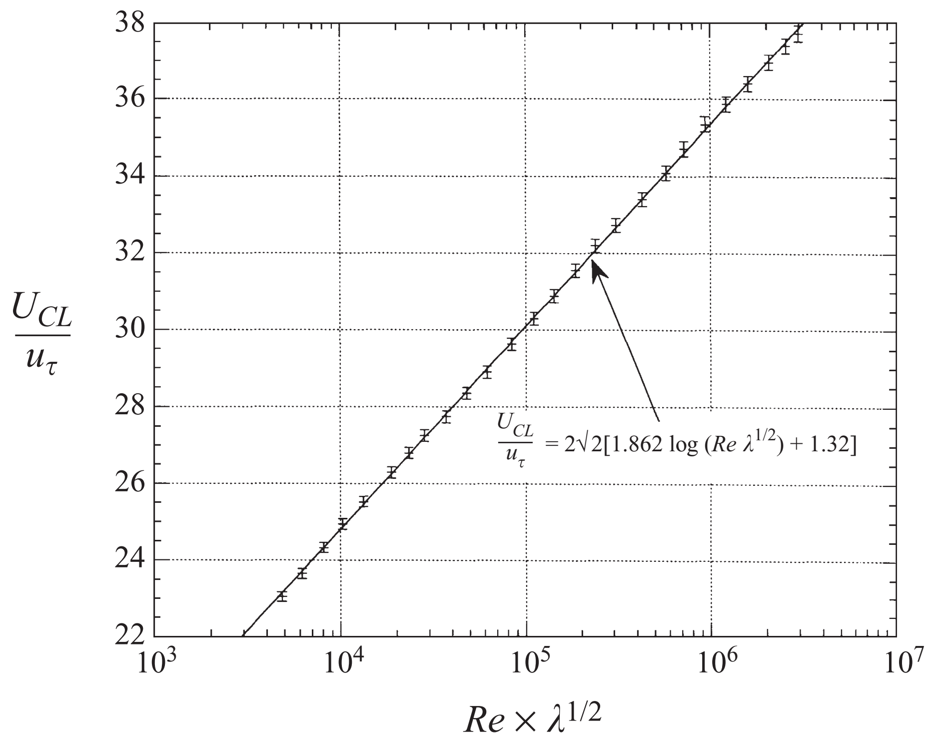
Merci beaucoup pour m’ écouter

Extra

Re vs Re_τ



38
36
34
32
30
28
26
24
22
$\frac{U_{CL}}{u_\tau}$
$\frac{U_{CL}}{u_\tau} = 2\sqrt{2}[1.862 \log(Re\,\lambda^{1/2}) + 1.32]$
10^3
10^4
10^5
10^6
10^7
$Re \times \lambda^{1/2}$