SECTION

TURBULENCE LIBRE

FREE TURBULENCE

Président : Professeur H. W. LIEPMANN

FREE TURBULENT FLOWS

Hans W. LIEPMANN
California Institute of Technology
Pasadena. California

1. INTRODUCTION

Le mouvement d'un fluide incompressible non visqueux initialement irrotationnel, et dans un champ de forces conservatrices, restera irrotationnel, excepté pour les surfaces de tourbillons concentrés.

Ces surfaces de discontinuité doivent provenir de parois solides.

Le champ d'études du comportement de ces surfaces de discontinuité dans un fluide réel peut être appelé le champ d'écoulement libre laminaire ou turbulent.

Dans une telle étude, l'effet des surfaces solides responsables des nappes tourbillonnaires est expliqué par les seules conditions initiales de volume; par exemple par le champ d'écoulement libre laminaire ou turbulent.

Ces derniers dépendront souvent à leur tour (mais pas toujours) du mouvement libre lui-même; mais l'étude de cette interaction est généralement considérée comme un domaine séparé, et les paramètres de volume sont obtenus par l'expérience si nécessaire.

Le nombre de Reynolds caractéristique d'un écoulement particulier est formé avec ces paramètres. L'écoulement laminaire libre est possible quelque soit le nombre de Reynolds.

Pour les nombres de Reynolds élevés, la théorie de la couche limite est applicable à l'écoulement laminaire libre à une distance suffisamment importante des surfaces solides génératrices.

L'écoulement turbulent libre évidemment se produit seulement pour des nombres de Reynolds élevés.

Il est remarquable, c'est-à-dire pas du tout évident, que les écoulements libres turbulents présentent aussi une sorte de comportement à couche limite, dans des conditions analogues à celles que prennent les écoulements laminaires libres.

Pour les écoulements laminaires ceci peut être présenté comme une conséquence des équations du mouvement. Pour un écoulement turbulent, c'est un résultat essentiellement empirique, et il est, quelques fois au moins, difficile de fournir un argument convaincant montrant que c'est un résultat possible, c'est-à-dire compatible avec les équations du mouvement.

Il existe suffisamment de preuves expérimentales pour montrer qu'il est possible de considérer la transition vers le régime turbulent séparément, c'est-à-dire qu'il semble possible, à un nombre de Reynolds suffisament élevé, de produire un écoulement libre turbulent qui ne dépend pas du processus de transition ou de localisation de la zone de transition.

Cet énoncé n'exclut pas bien sûr, l'existence d'écoulements expérimentaux pour lesquels la transition influe sur l'écoulement dans la région turbulente.

Les types les plus simples d'écoulements turbulents libres sont :

1) la région de mélange provenant d'une surface plane de séparation entre deux écoulements parallèles, de vitesses différentes, mais d'égale pression statique moyenne.
2) le sillage.
3) le « jet ».

Le premier cas pouvant être considéré comme un cas limite spécial soit de 2, soit de 3, pour un écoulement plan.

La différence essentielle entre les écoulements libres turbulents, et les couches limites turbulentes, est l'absence d'une sous-couche laminaire dans le premier cas. L'écoulement est libre en ce sens qu'aucun rotationnel nouveau n'est formé au cours de son développement.

On a cependant plus de raisons d'espérer pouvoir formuler des lois de similitude asymptotiques, dépendant seulement des paramètres de volume caractérisant l'écoulement. Il semble que ce soit le cas inconditionnel pour lequel on possède suffisamment d'expérience.

Je veux encore insister sur le fait que, même ces lois de similitude ne sont pas tellement évidentes et nettes pour qu'aucun problème ne subsiste.

1. Introduction

The motion of an inviscid incompressible fluid, initially irrotational and in a field of conservative forces, will remain irrotational except for surfaces of concentrated vorticity. These discontinuity surfaces must originate on solid boundaries.

The field of study of the behavior of these surfaces of discontinuity in a real fluid can be termed the field of free laminar and free turbulent flow. In such a study the effect of the surfaces which are responsible for the vortex sheets is accounted for by bulk initial conditions only; e. g., by the net forces, the net momentum discharge or the net total flux of vorticity. These in turn will often — but not always — depend upon the free motion itself, but the study of this interaction is usually considered a separate field, and the bulk parameters are if necessary obtained from experiment. The Reynolds number characteristic of a particular flow is formed with these bulk parameters. Laminar free flow is possible for any Reynolds number. For large Reynolds numbers, boundary layer theory is applicable to laminar free flow at a sufficiently large distance from the originating solid surfaces. Turbulent free flow occurs of course only at large Reynolds numbers. It is remarkable; i. e., not at all obvious, that turbulent free flows should also show a kind of boundary layer behavior under similar conditions as do laminar free flows. For laminar flow this can be shown as a consequence of the equations of motion; for turbulent flow this is essentially an empirical result and it is sometimes at least not easy to give a convincing argument that it is a possible result; i. e., compatible with the equations of motion.

There is sufficient experimental evidence available to show that it is possible to consider the transition to free turbulent flow separately; i. e., it seems possible at a sufficiently high Reynolds number to produce turbulent free flows which do not depend upon the transition mechanism or the location of the transition zone. This statement does not preclude of course the existence of experimental flows for which transition did affect the flow in the turbulent region.

The simplest types of turbulent free flows are : (1) the mixing region originating from the plane interface between two parallel flows with different velocity but equal mean static pressure; (2) the wake; (3) the jet. Case (1) can be considered a special limiting case of either (2) or (3) for plane flow.

The essential difference between turbulent free flows and turbulent boundary layers is the absence of a laminar sublayer in the former. The flow is free in the sense that no new vorticity is generated in the course of its development. One has more reason therefore to hope that asymptotic similarity laws, dependent only on the bulk parameters characterising the flow, can be formulated. This does seem to be the case under all conditions for which sufficient experiments are available. Still, I like to emphasize that even these similarity laws are not so obvious and clear-cut that no problems remain.

2. Similarity and mean-velocity profile

The simplest case which exhibits the similarity of the flow variables is the mixing zone. Let U_1 and U_2 denote the mean velocity at $x = -\infty$. A thin plate separates the streams up to $x = 0$, at which point the interface begins and extends toward $+\infty$; the interface lies in the xz plane.

If we assume that for sufficiently high Reynolds numbers the flow becomes completely independent of the viscosity, it must be conical; i. e., all dependent variables must be consant along rays from the origin and can depend only on the angle $\theta = \tan^{-1} \frac{y}{x}$. This is the usual and quite natural assumption for semi-empirical theories and for representing experimental results. However, strictly speaking, such a flow is not possible in steady, subsonic motion. It is impossible to match a uniform flow to a conical region. This does not seem to be serious, since one really does not expect the wedge-shaped mixing zone to extend to infinity. However, difficulties like this sometimes indicate an unduly pronounced influence of boundaries in experimental setups, or an unexpectedly strong interaction between the free layer and the flow upstream, or finally the necessity for non-stationary flow.

If we accept conical flow and a boundary layer form of the equation, we obtain a relation between the mean shear τ and the x component of the mean velocity u in the form

$$\frac{d\tau}{du} + \rho \int_{\eta_0}^{\eta} u \, d\eta = 0 \tag{1}$$

where $v(\eta_0) = 0$.

The two usual assumptions for the form of τ are :

$$\tau = \rho \left(\frac{du}{d\eta} \right)^2 \quad \text{Prandtl-Tollmien (Mixing length} \sim x) \tag{1a}$$

$$\tau = \beta \rho \left| U_1 - U_2 \right| \frac{du}{d\eta} \quad \text{Prandtl-Görtler (Exchange coefficient} \sim x) \tag{1b}$$

The resulting differential equation for u is in both cases of third order and hence requires a boundary condition on the normal component v which is not unique unless the boundary layer solution is matched to a solution of the equation at infinity. Different assumptions here tilt the wedge-shaped region about the origin; e. g., one could demand $v(0) = 0$, corresponding to the perturbation scheme about the undisturbed interface. However, Tollmien assumed $v(+\infty) = 0$, and Görtler $u(0) = \frac{U_1 + U_2}{2}$; none of these assumptions affect the comparison of the computations with the experiment enough to cause concern, but in principle the question is left open.

With $\tau \sim \left(\frac{du}{d\eta}\right)^2$, (1$a$), the velocity profile terminates with discontinuous second derivatives along two rays at finite angles; i. e., one obtains a quasi-hyperbolic result. The assumption $\tau \sim \frac{du}{d\eta}$, (1$b$), leads to a profile which approaches the uniform velocities asymptotically. These general differences between the results from the two standard forms of τ are the same if applied to the other types of turbulent free flow.

The behavior is easily demonstrated if one approximates to Eq. (1) by replacing u in the integral by the inviscid solution

$$u = U_1 + (U_2 - U_1)\,H(\eta)$$

The resulting equation is linear and easily solved. The velocity profile obtained is a third degree polynominal if (1a) is assumed, and an error function if (1b) is assumed.

The sharp cut-off which results if the original mixing-length concept is applied, together with a minor difficulty near $\frac{du}{d\eta} = 0$, was a cause of concern and led Prandtl in 1942 to propose the use of an exchange coefficient or eddy viscosity proportional to the width of the mixing zone; i. e., to a linear relation between shear and rate of strain. Both concepts, the mixing length and the constant exchange coefficient, were suggested from an analogy with the kinetic theory of gases. The former concept is obviously modeled after the mean free path concept, the latter is similar to an often useful heuristic description of rarefied gas flow; i. e., flow in which the mean free path becomes of the order of the macroscopic characteristic lengths of the problem.

The analogy with the theory of kinetic gases can be and has been criticized. It is however interesting to note that the discovery of the intermittent and double structure of free turbulent flows removes the objection raised against sharp cut-offs, and at the same time makes it clear that a description of the whole, time-averaged flow by a simple expression for τ cannot do justice to the physical problem. Still, the fact remains that the concept of an exchange coefficient, in particular, is highly successful in the application to free turbulent flows as well as in the outer part of the boundary layer. Relating the stress and the velocity gradient is mainly criticized because it assumes " gradient diffusion "; i. e., diffusion determined by conditions in the neighborhood of a point. While the concept of the exchange coefficient or the mixing length doubtless originated from a gradient-diffusion model, the resulting expression does not necessarily imply this. Indeed, Narasimha (1961) has recently shown that the process of collisionless diffusion of gas clouds into vacuum can be rigorously interpreted as a diffusion process of the gradient type with a diffusion coefficient proportional to time; i. e., precisely the same form of expression as used in Prandtl's second formulation of the free turbulent exchange.

Looking at the phenomenological approaches today, I feel I have to retreat somewhat from the position I held previously, namely that their time has passed. It now seems to me that, recast and reinterpreted in relation to recent results both in turbulence and in the theory of fluids, they still have a future.

3. Intermittency

The single most important experimental finding in turbulent shear flows is the discovery of the existence of coherent regions of nearly homogeneous vorticity fluctuations separated from the rest of the fluid by sharply defined interfaces. In free turbulent flows this phenomenon is usually termed " intermittency ", following Townsend. In this sense it was first observed by CORRSIN (1943); however its importance and significant features were only recognized later by TOWNSEND (1949) and CORRSIN and KISTLER (1954). The consequences of the observations can be explored in two directions. The intermittent occurrence of turbulent regions near the edge of a shear flow, besides altering the evaluation and interpretation of measured mean values of the flow parameters, demonstrates a large scale superstructure imposed upon a fine scale turbulent flow. It is clear that intermittency is a feature which has to be incorporated into any theory of free turbulence.

On the other hand, the existence of the sharp interfaces itself presents a very interesting and far-reaching problem. It is indeed quite common and occurs in Charters' zones of contamination (CHARTERS, 1943), in Emmons' spots (EMMONS, 1951), in the turbulent slugs studied by ROTTA (1956) in pipe flow, and in the spiral turbulence of COLES (1960).

4. Turbulent interfaces

The motion of turbulent interfaces and the fluid flow in the neighborhood of such interfaces stands out as a relatively new and by no means solved problem in turbulence. The subject will be discussed in a special lecture by Prof. COLES. Here I will stress only a few points of immediate interest to free turbulent flows.

First of all, one may formulate a general problem as follows : Consider a flow field $\vec{u}(\vec{x}, t)$ such that $\vec{u} = \text{grad}\, \phi$ at $y = +\infty$; $\vec{\xi} = \text{curl}\, \vec{u}$ is a stochastically known function at $y = -\infty$; e. g., $\overline{\xi^2}$, $\overline{\xi(\mathbf{x}, t)\, \xi(\mathbf{x} + \mathbf{v}, t + \tau)}$ etc. are known and independent of t. How much can be said about a flow field with these properties and what are the restrictions for its existence ? In particular, experience shows that the region of vorticity fluctuation is bounded by a sharply defined surface. How does the existence and motion of such a surface follow from the equations of motion ? For example is a mean motion necessary ? Is a mean vorticity field necessary ?

To my knowledge little if anything is known about the general problem, which by the way bears a striking, but superficial, similarity to certain problems of shock waves in transonic flow.

As in the shock wave problem, one may first study the properties of the flow in the neighborhood of a given plane interface. Turbulent-non turbulent interfaces have been observed in pipe flow, where they are essentially normal to the direction of mean motion, and in intermittent free turbulent flows, where they are essentially parallel to the mean flow. Here I will mainly discuss the latter in relation to intermittent turbulent flow.

The simplest model to be discussed assumes a statistically homogeneous field of vorticity fluctuations for $x_2 < 0$ which is separated by a plane interface in the x_1, x_3 plane from irrotational fluid at $x_2 > 0$. There must exist velocity fluctuations induced by the random vorticity field, in the irrotational flow as well. The irrotational velocity fluctuations are thus statistically homogeneous in any plane $x_2 =$ constant and, as CORRSIN and KISTLER (1954) have shown, this leads to a number of general relations for the mean values of the irrotational velocity fluctuations and their derivatives. In irrotational flow of an incompressible fluid

$$\operatorname{div} \vec{u}\,\vec{u} = \operatorname{grad} \frac{|u|^2}{2}$$

Hence the forces due to the Reynolds stresses are equivalent to a pressure gradient; applied to fluctuations which are statistically homogeneous in x_1 and x_3, this implies

$$\frac{\partial}{\partial x_2} \overline{u_2^2} = \frac{\partial}{\partial x_2} \frac{|u|^2}{2}$$

or

$$\overline{u_2^2} = \overline{u_1^2} + \overline{u_3^2}$$

To obtain the details of the irrotational fluctuations one has to solve the equation $\nabla^2 \phi = 0$ for given stochastic boundary conditions in the plane $x_2 = 0$; i. e., one has to solve $\nabla^2 \phi = 0$ for an upwash distribution $v(x_1, x_3, t)$. This problem was solved by PHILLIPS (1955) using a spectral representation of the upwash field. The solution for the velocity fluctuation of course agrees with the general behavior noted above and in addition shows explicitly the decay of the fluctuation with distance from the interface. This decay is algebraic and of the form $\overline{\vec{u}\vec{u}} \sim x_2^{-n}$. The exponent n depends upon the conditions imposed on the fluctuations at the interface. For example, if the total source strength is assumed to vanish, then $n \geq 4$. This can be seen easily in the following way: the irrotational velocity fluctuations $\vec{u}(\vec{x}, t)$ are related to the upwash field $v(\vec{s}, t)$ by Rayleigh's formula

$$\vec{u} = \frac{1}{2\pi} \int d\vec{s}\, v(\vec{s}, t) \frac{\vec{r}}{r^3}$$

where $\vec{s}$ is the position vector in the $x_2 = 0$ plane, and $\vec{r} = \vec{x} - \vec{s}$. The tensor $\overline{\vec{u}\,\vec{u}}$ is then related to the correlation function $\Psi(\vec{l})$ of the upwash distribution by

$$\overline{\vec{u}\,\vec{u}} = \frac{1}{4\pi^2} \iint d\vec{s}\, d\vec{s}' \cdot \Psi(\vec{l}) \frac{\vec{r}\,\vec{r}'}{r^3\, r^{3\prime}}$$

with $\vec{l} = \vec{s} - \vec{s}'$. To obtain the asymptotic behavior of $\overline{\vec{u}\,\vec{u}}$ at large distances from the interface, develop Ψ into a series of δ functions;

$$\Psi(\vec{l}) = \overline{v^2} \left[\mathrm{A}\, \delta(\vec{l}) + \mathrm{B} \cdot \frac{\partial \delta}{\partial \vec{l}} + \Gamma \frac{\partial^2 \delta}{\partial \vec{l}\, \partial \vec{l}} + \dots \right]$$

The vector B is zero because of symmetry, $\Psi(\vec{l}) = \Psi(-\vec{l})$; A vanishes if one assumes zero total source strength. The function Ψ is then determined by the tensor Γ which is proportional to the moment of inertia of the correlation ellipse of the upwash. The quantity Γ has the dimensions (length)^4, hence $\overline{\vec{u}\,\vec{u}} \sim \overline{v^2}\,\Gamma\, x_2^{\,-4}$. The character of v is dipole-like; the mean square of the components of $\vec{u}$ is given by a superposition of dipole distributions.

The complete form of $\overline{\vec{u}\,\vec{u}}$ at large distances is given by

$$\overline{\vec{u}\,\vec{u}} = \frac{\overline{v^2}}{4\pi^2} \int d\vec{s}\,\Gamma \left(\frac{\partial}{\partial \vec{s}} \frac{\vec{r}}{r^3} \right) \left(\frac{\partial}{\partial \vec{s}} \frac{r}{\vec{r}^3} \right)$$

If the coordinates x_1 and x_3 coincide with the principal axes of the correlation ellipse, $\overline{\vec{u}\,\vec{u}}$ is diagonal, i. e. no Reynolds stresses are produced in the potential flow as discussed by Stewart (1956).

All these computations are quite straight-forward : however, the choice of realistic assumptions for Ψ and the comparison with experiments are much more difficult. It is not obvious, for example, that the source strength can always be chosen zero. The large scale motions of the interface as encountered in experiments allow only a comparison with theory close to the moving surface if only fluctuations in the potential region are measured, or very far away from the moving surface. This simply reflects the existence of two different scales in the fluctuations. The interface can be considered plane at distances large compared to the large scale motion, or close by, where the curvature due to the large scale motion is negligible. The applicability of the asymptotic results to the latter requires that the largest scale of the small-scale motion is still much smaller than the smaller scales in the « big-eddy » motion. This is a stringent requirement and I believe that the agreement between the computations and Townsend's measurements in the wake quoted by Phillips is largely fortuitous. One may add here that a random whipping of the wake as a whole contributes a sources term in the analysis and hence has a pronounced effect on the measurements.

The matching of a potential flow field to an upwash distribution on a plane or slightly curved surface is in essence equivalent to the lumping together of the vorticity-fluctuation effects into a source sheet, which then represents a fluctuating displacement thickness. This approach is very useful in describing the flow at large distances both in the potential fluctuation problem and the corresponding acoustic problem (Liepmann, 1954), but it is not capable of describing the fluctuation near the bulges of the sloshing interface. The velocity field close to the interface can be obtained, at least in principle, by computing the induced velocity field from the vorticity fluctuation in the volume bounded by the interface. The velocity is then given by

$$\vec{u} = \operatorname{curl} \frac{1}{4\pi} \int d\vec{s}\, \frac{\vec{\xi}(\vec{s}, t)}{r}$$

where $\vec{\xi}$ denotes the instantaneous vorticity at a point with radius vector $\vec{s}$, with $\vec{r} = \vec{x} - \vec{s}$.

The vorticity $\vec{\xi}$ is nearly statistically homogeneous on one side of the sloshing interface and zero on the other. This behavior can be expressed by writing for $\vec{\xi}$

$$\vec{\xi} = \vec{\omega}(\vec{s}, t)\, \mathrm{H}(\vec{s} - \vec{\sigma})$$

where $\vec{\omega}$ is the homogeneous vorticity fluctuation, H the Heaviside function, and $\vec{\sigma}$ a random vector.

The velocity correlation can thus be written :

$$\overline{\vec{u}\,\vec{u}} = \frac{1}{16\pi^2} \iint d\vec{s}\, d\vec{s}'\, \overline{\frac{\vec{\omega} \times \vec{r}}{r^3}\, \frac{\vec{\omega}' \times \vec{r}'}{r'^3}\, \mathrm{H}\,\mathrm{H}'}$$

The position of the interface and the field $\vec{\omega}$ will be nearly statistically independent, and hence the quadruple correlation in the integral splits into a product

$$\overline{\vec{u}\,\vec{u}} = \frac{1}{16\pi^2} \iint \frac{d\vec{s}\, d\vec{s}'}{r^3 \cdot r'^3}\, \overline{(\vec{\omega} \times \vec{r})\, (\vec{\omega} \times \vec{r}')}\ \overline{\mathrm{H}\,\mathrm{H}'}$$

The integral formally expresses the remarks made above. There are two scales involved, λ and Λ, say, corresponding to the ω and H correlation respectively. Hence there is the possibility for *two* far field cases, i. e. for $\lambda \ll r \ll \Lambda$ and $\Lambda \ll r$.

One can simplify the integral by noting that the switching on and off of the vorticity field, expressed by H, has the character of Rice's " random telegraph signal ". The correlation function can then be represented by

$$\overline{\mathrm{H}\mathrm{H}'} = \frac{1}{2} \exp\left\{ - \frac{(\vec{s} - \vec{s}') \cdot \vec{n}}{\Lambda} \right\}$$

where $\vec{n}$ is the unit vector normal to the mean interface plane. (Corrsin and Kistler (1954) have made very similar assumptions and provided a partial experimental check on them.) For the far field one can again use development in δ functions like the one used in the discussion of Phillips' model; even so, the detailed expressions become rather messy because the $\vec{\omega}$ correlation involves fourth derivatives of the δ function.

A final remark may be made concerning the mean flow near the sloshing interface. The velocity within the bulges but outside the interface must be potential, but not necessarily parallel and equal to the flow at infinity as assumed by Corrsin and Kistler. Stewart (1954) has pointed this out correctly; his detailed sample computation on this point, however, is not convincing, since it stretches a linearized solution too far. The interface is certainly not a stream- or path-surface, since the growth in width of any shear flow requires fluid entrainment. Hence the « jump condition » for particles crossing the interface becomes important.

5. The stability of " turbular " fluids

It is very tempting indeed to consider the fine scale turbulent motion in a particular fluid, call it « turbular », in the light of the observed large scale structure and instability of the motion of such a fluid. I mentioned a point of view like this in a survey paper about 10 years ago (Liepmann, 1952), but recent work, e. g. Roshko's observation of a Karman vortex street at a Reynolds number of ten million (Roshko, 1961) tends to confirm the usefulness of such a heuristic view. Furthermore, whenever a stability computation of a laminar shear flow has been made, the ratio of critical wave length to width of the shear region has turned to be large (of the order of ten) regardless of the details of the shear region. This fact again suggests the possibility of a separate treatment of the large-scale motion and its relative lack of sensitivity to the details in the shear region. Therefore I want to dwell on this point in somewhat more detail.

First of all it is clear that in searching for oscillations capable of representing intermittency or vortex-street behavior in a turbulent flow one must end up with a model somewhat like a non-linear oscillator with a limit cycle. The damping of any perturbation and/or the amplification must be a function of the perturbation amplitude or energy. In the only detailed model of free turbulent flow, the " big eddy structure " of Townsend, this has been attempted and, for the particular model chosen, accomplished. However, I want to stress the relation to ordinary laminar instability a little more and to emphasize the details of the large-scale structure less.

In the present interpretation, Townsend's approach is very closely related to some early work on laminar instability by the use of the energy method. The stability problem is determined by the ratio of two integrals, often denoted by M and N;

$$M = -\int dV \, \rho \, \overline{\vec{u}\,\vec{u}} \cdot \text{grad}\, \vec{U}$$

$$N = \int dV \, \overline{\vec{\tau} \cdot \text{grad}\, \vec{u}}$$

Here $\vec{u}$ denotes the perturbation velocity field, $\vec{U}$ the mean flow field, and τ the viscous shearing stress.

The ratio $\frac{M}{N}$ is proportional to a characteristic Reynolds number of the flow, R. The motion becomes unstable when $\frac{M}{N}$ reaches unity and this condition determines the critical Reynolds number. Numerical values of such computations have been obtained for a few simple cases of two-dimensional flow. The critical Reynolds numbers obtained are low, of the order of 100 or less.

Translating the same approach to the turbular fluid, we have only to interpret τ as the apparent shearing stress due to the small scale motion; indeed, by splitting the velocity field into three contributions U, v and q, say, one can obtain the result formally. (The form differs slightly from the one used in Townsend's book.) If we now use the exchange-coefficient concept it is clear that the stability criterion will simply lead to a value for R; i. e., a Reynolds number based upon the exchange coefficient and the local width of the free turbulent zone.

If the integrals are evaluated on the basis of a particular choice for the perturbation motion one can obtain a numerical value, somewhat dependent on the particular perturbation chosen, for the constant. Here as in the corresponding laminar case the problem should be handled as a variational one. It is however interesting and, I believe, not entirely fortuitous that the values obtained by Townsend are of the same order of magnitude as the laminar ones. However, for the turbular field we need more; namely, we have to limit the amplification. In detail, of course, this is a very complex non-linear interaction due to the dependence of the correlations and therefore of the apparent shear on the velocity derivatives of the larger-scale motion. Thus — within the framework of a heuristic model — the turbular fluid is non-Newtonian; the viscosity increases with the rate of strain and is not a simple scalar.

It appears interesting to carry the concept of such a fluid still further. For example, the extension of concepts like the ones used in irreversible thermodynamics may serve to define a quasi-entropy and a quasi-temperature. These concepts should be intimately related to the general question of irreversible energy transfer to the turbulent motion; i. e., what are the conditions for which τ grad $\vec{u}$ is positive definite. If τ is the ordinary shearing stress this is obviously so as long as μ is positive. For turbulent flow it is not necessarily true in general and it is quite an interesting problem to inquire into possible conditions for which τ grad u can be negative; i. e., for which energy is fed back from the fluctuating into the mean motion. Cases like this seem to require more organized fluctuations like the modes in the early stages of laminar instability. Conversely, one may be able to define " true " turbulence; i. e., the random energy content of such a turbular fluid, precisely by an irreversibility condition requiring the positive definite nature of terms like τ grad $\vec{u}$. One can find here some similarities with the behavior of matter at low temperatures, but it seems premature to discuss relatively unfounded speculations here.

Experiments have shown that the mean-temperature profiles in free turbulent flow are always wider than the corresponding velocity profiles. Both the spreading of heat and the spreading of momentum must be limited by the position of the interface, since the effects due to viscosity and heat conduction are too small in the Reynolds-number range of interest. Hence the mean-temperature profile can be wider than the mean velocity profile only as a result of effects within the nearly homogeneous part of the turbulent flow. The faster spreading of heat compared to momentum is thus a property of the " turbular fluid ". The characteristic differences in the ratio of heat diffusion to momentum diffusion found in plane flows compared to axially-symmetric flows demonstrate the difference between the transport of a scalar and a vector, a fact first shown explicitly by G. I. Taylor in his vorticity-transport theory. The experimental results here are very important as a basis for attempting to formulate a phenomenological transport theory of the "turbular fluid " in general.

6. The importance of the early stage of turbulent shear flows

The discussion has so far been restricted almost entirely to the asymptotic behavior of free turbulent flows, the region far downstream where the flow is similar (or self-preserving). Except for some (probably) minor problems noted before, such flows are compatible with the equations of motion and certain features agree well with the available experiments. There exist experiments, and also general considerations, which tend to show that the large-scale fluctuating motion in turbulent shear flows does depend on the early stage of development. The measurements of Roshko (1954) on the development of turbulent wakes from vortex streets and some recent work by Grant (1958) suggest the importance of the vortex street for the large-eddy structure or intermittency in the fully developed wake. Quite recently KISTLER (1961) has shown that even turbulence decay behind a grid depends markedly on the wake-producing bodies making up the grid. All these results are not quite unexpected, since it follows quite generally that changes in any large scale structure in turbulent flow are slow (e. g. BATCHELOR, 1953); still, the term " slow " has been made much more precise by these experiments, and the disappearance of the original organized structures has been shown to be very slow indeed. In terms of the instability picture this means that the original organized motion, such as a vortex street, leads to a far more selective choice of possible unstable modes than pure random formation of large-scale structures. Experiments here would be very important indeed. Comparisons of three-dimensional and two-dimensional wakes have not been made in detail. Two-dimensional wakes, with and without the suppression of the original vortex street by the use of a splitter plate, should also be studied. Similarly, it should be worthwhile to trace the relationship of the organized fluctuations existing in jets near the exit (WILLE, 1958) to the fully developed flow downstream. A re-study of the mixing zone from this point of view is also worthwhile, because here we deal with a single vortex sheet only. To conclude this particular section, I want to discuss " mathematical experiments " on vortex-street formation performed recently by ABERNATHY and KRONAUER (1961)*. Two plane vortex streets a distance h apart are

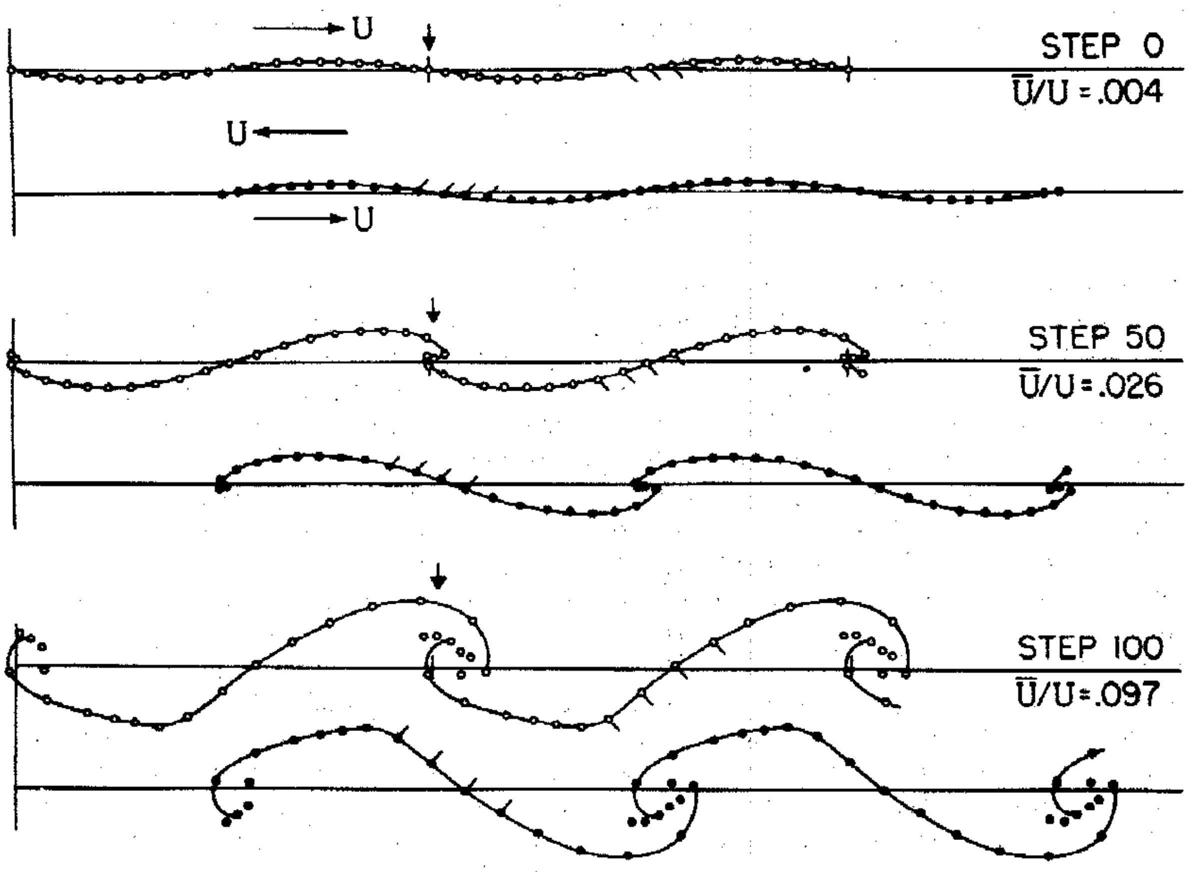


FIGURE 1

* I am much indebted to DRS. ABERNATHY and KRONAUER for supplying me with these figures from their unpublished work.

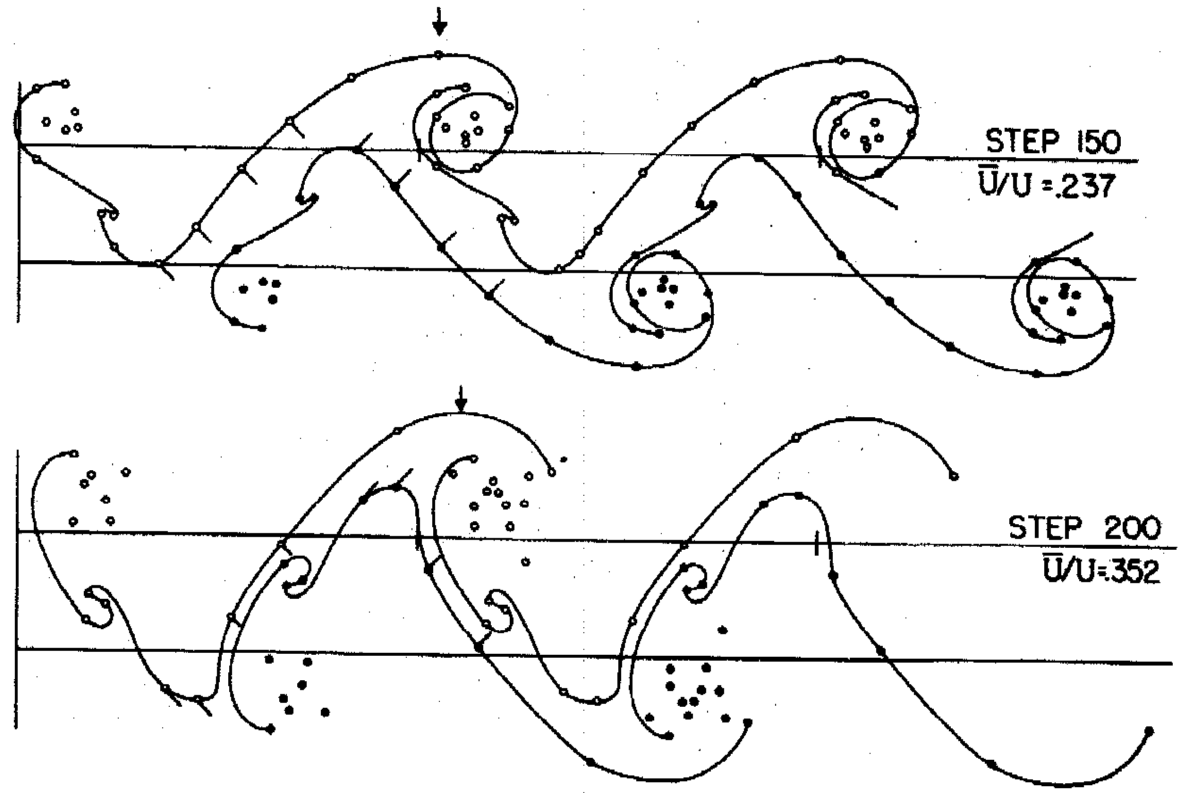


FIGURE 2

represented by a finite number of potential vortices. These vortices are perturbed by an antisymmetrical wave of wavelength a at $t = 0$, and the subsequent motion is followed in detail on a digital computer. This procedure if necessary represents mathematical experimentation on Karman's model of vortex streets and vortex-street stability. The motion depends strongly on the ratio h/a — in agreement with Karman's original theoretical work — and I find it fascinating and highly instructive simply to study the resulting occurrence of what Abernathy and Kronauer call " clouds " of vorticity. The first set of figures (Figs. 1 to 3) shows the development of the " street " for Karman's " stable " ratio of $h/a = 0.28$. The increase of the distance between the center of gravity of these clouds of vorticity and the dispersion of vorticity around the C. G. is evident — and this without viscosity ! Furthermore, the resulting confusion of the separate vortices is so large that after a while it is impossible to trace a distorted sheet through subsequent vortices, even with only 21 vortices per wavelength. Another sample of these experiments is shown in Fig. 4, where the ratio h/a has been chosen smaller; $h/a = 0.17$. Here the clouds of vorticity form much faster — as expected from Karman's analysis. However, another striking feature is the tendency to produce additional small clouds of vorticity near the center, between the original sheets. Note also the tendency for one vortex sheet to be wrappd around the other, resulting in the appearence of both positive and negative vorticity within the same limited cloud. The similarity of this computed model and observations by Homan is shown in Fig. 5. The similarity of the pattern to shadowgraphs of turbulent wakes is also quite obvious.

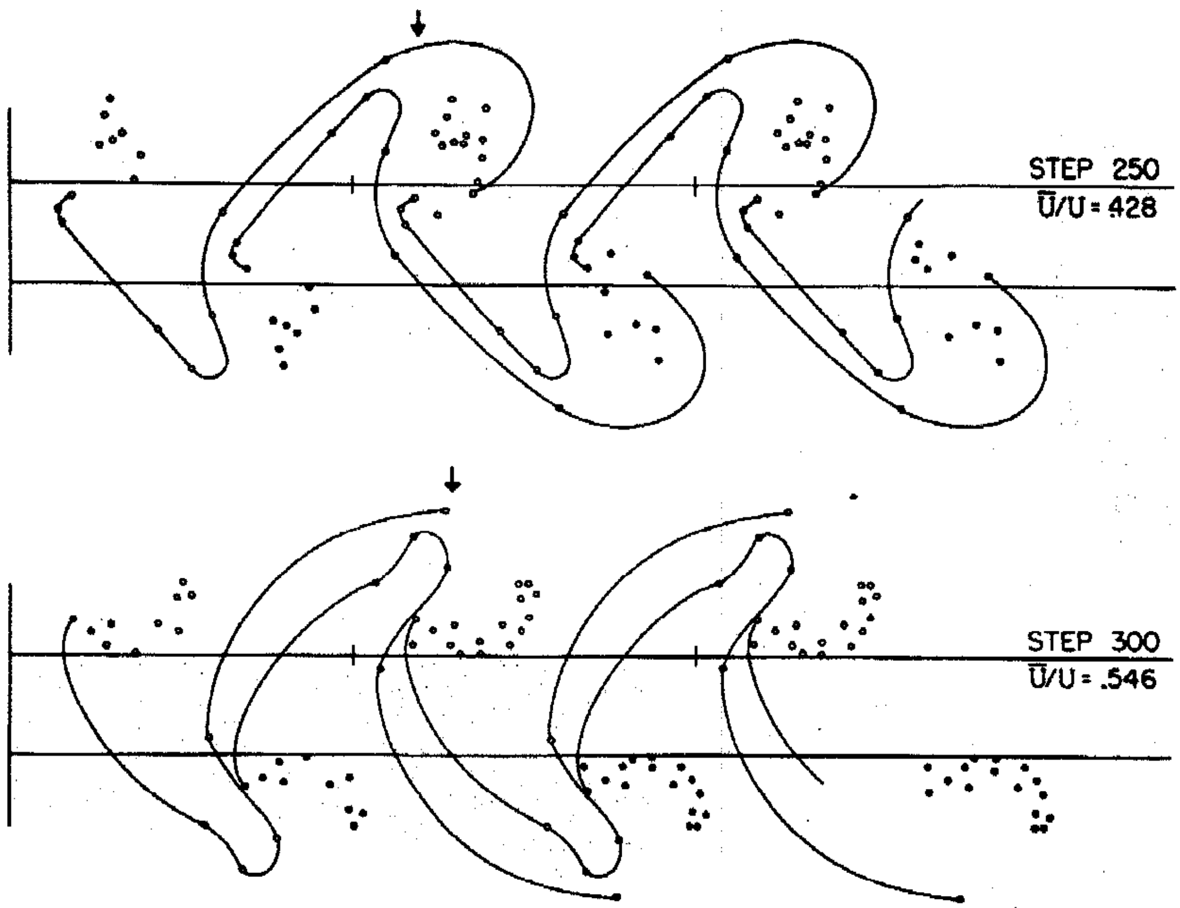


FIGURE 3

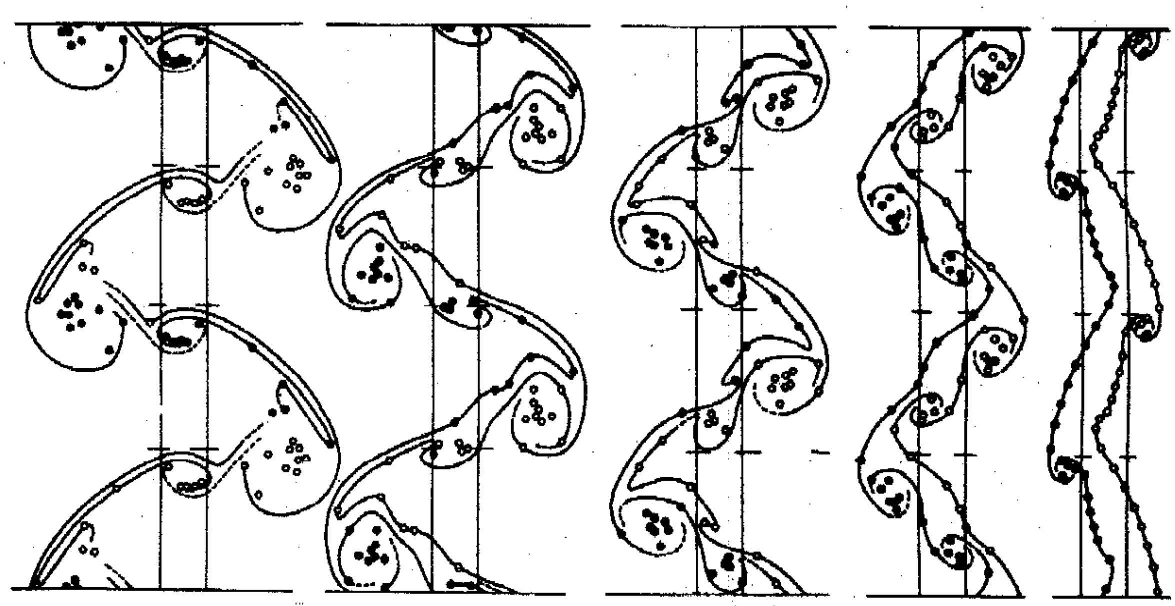


FIGURE 4

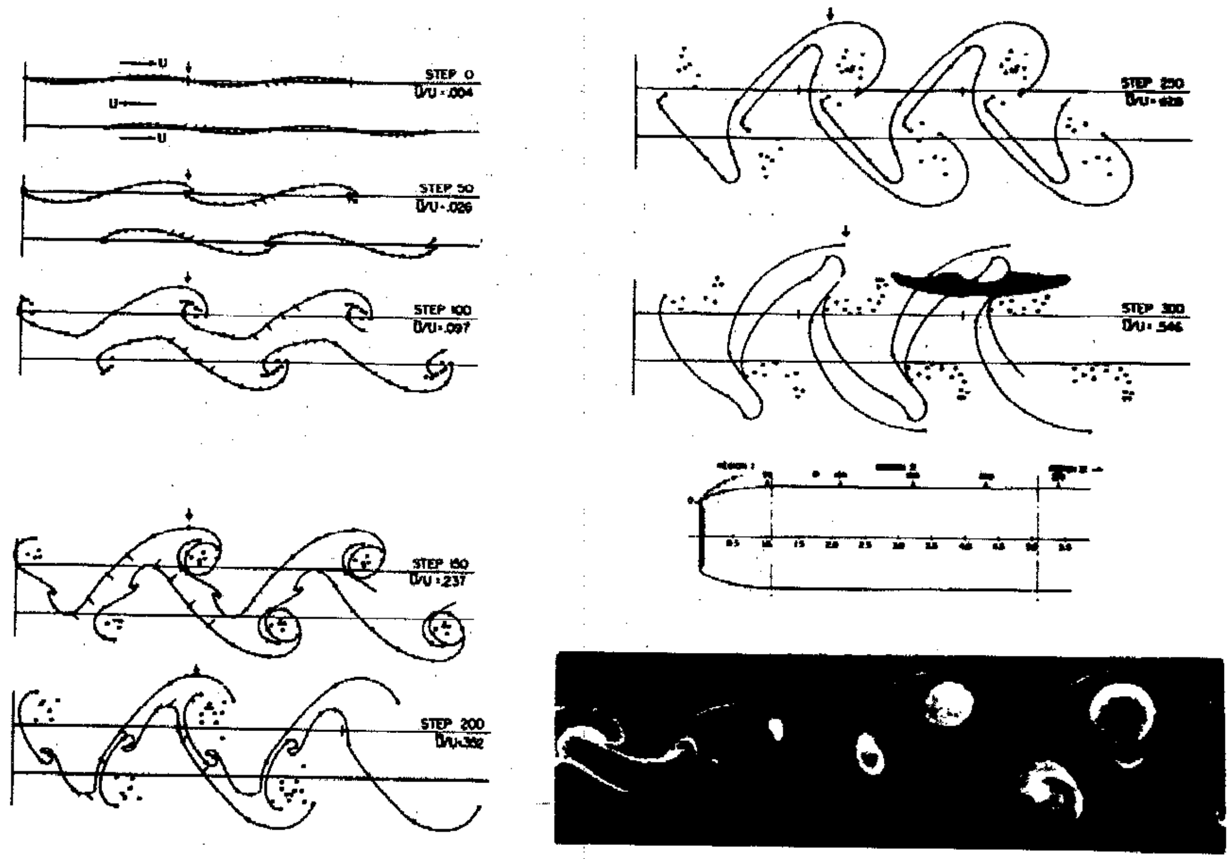


FIGURE 5

These patterns, which already show a nearly random small-scale structure within an organized large-scale structure, result from nothing more than a relatively small number of line vortices attempting to rotate about each other. Remembering that two-dimensional vortex motion is so much simpler than general vortex motion, one gets here a fascinating glimpse into rapid randomization by vortex interactions and a demonstration of the statistical character of turbulence proper !

7. Detailed description of the " large eddies "

I want to add here a few remarks concerning the choice of definite structures to describe the large-eddy motion. Attempts to do so are of course not new. It always looks attractive to consider at least part of the turbulent flow as an ensemble of definite structures. Synge and Lin, for example, discussed in detail an ensemble of Hill vortices to represent turbulent flow. Theodorsen introduced the horseshoe vortex as the standard element of turbulence, and Townsend and Grant have attempted to fit a particular eddy structure to the measured two-point correlation function in wake and boundary-layer flows.

These models all have their use in focusing attention on certain basic phenomena; in particular, the fundamental phenomenon of vortex stretching can be followed in some detail with such models. I do not believe, however, that these models will ultimately remain in any final theory of turbulence. The success of the spectral representation of turbulent fields is due, after all, not to the belief in the existence of definite waves but to the possibility of representing quite general functions as Fourier integrals. In the application to stochastic problems the usefulness of the Fourier representation stems essentially from the translational invariance.

Consequently, really successful models for representing turbulent shear flows will require far broader invariance considerations. It is clear that the essence of turbulent motion is a vortex interaction. In the particular case of homogeneous isotropic turbulence this fact is largely masked, since the vorticity fluctuations appear as simple derivatives of the velocity fluctuations. In general this is not the case, and a Fourier representation is probably not the ultimate answer. The proposed detailed models of an eddy structure represent, I believe, a groping for an eventual representation of a stochastic rotational field, but none of the models proposed so far has proven useful except in the description of a single process.

8. Remarks on experiments

In a field like turbulence it is still difficult of impossible to define uniquely the decisive parameters to be measured accurately. Consequently, experiments on turbulence carried out without a point of view or working hypothesis are most often useless, since it is more than likely that immaterial and non-universal parameters will be measured. Because of this state of affairs it is unlikely that a definitive set of experiments for any flow configuration can be made once and for all. Measurements on the same configuration have to be repeated occasionally when a new idea shifts the point of view. The experiments on free turbulent flow which have been made so far reflect both a changing emphasis and a development of new techniques. In the early experiments the mean-velocity distribution and its relation to the mixing-length theories was the primary interest. The next set of experiments attempted a clarification of the exchange parameters and of the turbulent energy balance. The discovery of intermittency and of turbulent fronts during this time removed much of the basis for the comparison in the first stage, but did set the scene for the next one. It seems to me that the double structure of the turbulent flow is basic for future experiments. We have to deal now with a nearly homogeneous but non-isotropic small-scale turbulent field, with a large-scale motion which has the character of long instability modes, and with interfaces. There are at least a few problems for which these three patterns can be explored separately. For example, the development of Reynolds stresses and of turbulent heat fluxes in strained homogeneous turbulence is a problem which requires much further work; in the terminology used above, it amounts to an experimental determination of an equation of state and of transport parameters for the turbular fluid. It is quite possible that such a study would require very high Reynolds numbers to be significant.

In a study of the large-scale motion it seems to me definitely worthwhile to include the growth and decay of artificially induced perturbations. The Schubauer-Skramstad experiments have taught us this necessity for a study of instability in laminar flow. The same idea, but not necessarily the same technique, should be applicable to turbulent shear flows.

In the following lecture Prof. Coles will attempt to give a coherent representation of observations on interfaces under various flow conditions. The necessity for studying interfaces as a separate, highly interesting ingredient of turbulent flows seems to me clearly demonstrated in his contribution.

REFERENCES

Abernathy, F. and Kronauer, R. (1961). *J.F.M.*, in print.

Batchelor, G.K. (1953). *Homogeneous Turbulence*, Cambridge.

Charters, A. (1940). *N.A.C.A.* TN 831.

Coles, D. (1960). X Inter. *Congress of Theoretical and Applied Mechanics*, Stresa.

Corrsin, S. (1943). *N.A.C.A.* W-94.

Corrsin, S. and Kistler, A. (1954). *N.A.C.A.* TN 3133.

Emmons, H.W. (1951) *J.A.S.*, **18**, 490.

Grant, H.L. (1958). *J.F.M.*, **4**, 149.

Kistler, A. (1961). *J.F.M.*, in print.

Liepmann, H.W. (1952). *Z.A.M.P.*, **3**, 321-42 and 407-26.

Liepmann, H.W. (August 1954). *GALCIT* report to *N.A.C.A.*

Narasimha, R. (1961) *J.F.M.*, in print.

Phillips, O.M. (1955). *Proc. Cam. Phil. Soc.*, **51**, 220.

Roshko, A. (1954). *N.A.C.A.*, Rep. 1191.

Roshko, A. (1961). *J.F.M.*, **10**, 345.

Rotta, J. (1956). *Ing. Arch.*, **24**, 258.

Stewart, R.W. *J.F.M.*, 1, 593.

Townsend, A.A. (1956). *The Structure of Turbulent Shear Flow*, Cambridge.

Wille, R. (1958). In " Grenzschicht Forschung ", *IUTAM Symposium Proceeding*, Berlin.

DISCUSSION

Professor I. Tani. — In connection with the large-eddy problem, I would like to refer briefly to our recent observations on the boundary layer along a concave wall. The mean velocity measurements reveal that the spanwise variation has a definite wave number whether the boundary layer be laminar or turbulent. The variation is interpreted as produced by a system of vortices with axes in the streamwise direction. For the laminar boundary layer the vortices are identified with those predicted by the theory of laminar instability. For the turbulent boundary layer the concept of eddy viscosity is introduced to account for the observed phenomena by the theory of laminar instability. In both cases, however, the mechanism by which the wave number is determined remains an open question. Since the centrifugal force in the concavely curved boundary layer is analogous to the buoyant force in a thermally stratified layer in giving rise to instability, it is to be expected that some analogous phenomena should be observed in the boundary layer along a heated horizontal wall.

Professor J. Kestin. — I suspect that Professor Liepmann's suggestion that a Newtonian fluid endowed with a turbulent structure could be regarded as a non-Newtonian fluid in its dynamic behavior, may prove to be very fruitful. If this idea is accepted as a tentative, working hypothesis, it is clear, as Professor Liepmann so eloquently demonstrated, that many exciting avenues for exploration would become apparent.

The analogy in the behavior of a non-Newtonian fluid and Professor LIEPMANN's "turbular" fluid has been observed, and exploited first, as far as I can ascertain, by Professor RIVLIN (RIVLIN, R.S. (1957). The relation between the flow of non-Newtonian fluids and turbulent Newtonian fluids. *Q - ly Appl. Math.*, **15**, 212). In particular, Professor RIVLIN stressed the analogy between secondary flows, for example in elliptic pipes, which occur with non-Newtonian fluids and in turbulent flows, and the possibility of the occurrence of so-called normal stress effects in turbulent flows.

He concluded his paper with the statement :

" ... it appears quite likely that a phenomenological theory of the type considered, in which the stress in an element of the turbulent fluid (large compared with the eddy dimensions) is supposed to depend only on the kinematic variables in that element or on the velocity-gradient history of that element, will be entirely adequate as a complete description of the flow properties of the turbulent fluid, since eddies can diffuse from one point of the fluid to another ".

Mr. P. S. KLEBANOFF. — With respect to the comment made as to the experimentally observed KARMAN-TAYLOR vortices in turbulent flow, and the inference of instability of turbulent flow, one has to allow for the possibility that such motions may result from the laminar instability process, or may be due to irregularities in the wind stream.

INTERFACES AND INTERMITTENCY IN TURBULENT SHEAR FLOW

Donald COLES
California Institute of Technology
Pasadena - California - U.S.A.

SOMMAIRE

On passera en revue des expériences sur des écoulements intermittents dans un tube, des écoulements de Couette circulaires, des écoulements de couche limite, et des écoulements libres complètement établis dans des sillages ou des jets. L'accent a été mis sur l'aspect descriptif plutôt que sur l'aspect analytique du problème. Ce rapide survol de la question indique clairement qu'il est nécessaire d'effectuer une étude plus poussée de la stabilité des écoulements mixtes (laminaire-turbulent) au cours du régime de transition, de la propagation de l'interface dans des conditions très générales, et des problèmes qui s'y rattachent.

SUMMARY

Experiments on intermittency are reviewed for pipe flow, circular Couette flow, boundary-layer flow, and fully developed free shear flow in wakes and jets. Phenomenological rather than analytical aspects of the problem are emphasized. The survey clearly indicates a need for further work on the question of stability for mixed laminar-turbulent flows in the transition regime, on the question of interface propagation in a general environment, and on related questions.

A. — Introduction

The phenomenon of intermittency was first recognized as an important feature of turbulent shear flow about twenty years ago, and has since been encountered in many different experimental situations. The main current of theoretical research in turbulence, however, has not dealt directly with this particular problem, and the available experimental information is widely scattered in the literature. In keeping with the spirit of this colloquium, the purpose of this article is to review existing knowledge of the subject, to identify as far as possible the important physical processes connected with intermittency, and to suggest areas in which further research is needed.

At the outset this survey was intended to be a discussion of intermittency as it occurs near the boundaries of free shear flows such as the wake, the jet, and the mixing layer. For several reasons, however, it has become necessary to include a discussion of intermittency as it occurs during transition, especially in pipe flow and circular Couette flow. In all cases, intermittency implies the existence of definite interfaces separating regions of laminar and turbulent flow. The characteristic scale, propagation velocity, and other properties of these interfaces appear to be determined primarily by energy processes involving the largest eddies in the turbulent regions. These energy processes in turn are sometimes quite sensitive to changes in Reynolds number, at least in the neighborhood of the lowest Reynolds number for which shear turbulence can maintain itself in any given environment. From this point of view the flow in a transition region has the advantage that both the energy balance and the interface geometry are highly exposed, experimentally speaking. Moreover, experiments related to intermittency can be carried out under relatively closely controlled conditions in a transition region, in contrast to the situation in a fully developed free shear flow.

B. — Pipe Flow

Transition in pipe flow is marked by intermittent turbulence which takes the form of alternating slugs of laminar and turbulent fluid moving down the pipe. The transition range in terms of Reynolds number $R = \frac{Ud}{\nu}$ (where $U =$ mean velocity, $d =$ pipe diameter) can readily be identified by observing the flow far downstream for highly disturbed entry conditions. For $R < 2000$ (approximately), turbulent regions tend to decay and disappear as the fluid proceeds downstream; for $R > 2800$ (approximately), turbulent regions tend to spread into laminar ones and to fill the pipe completely. In an intermediate range $2200 < R < 2600$, however, there apparently exists a regime of mixed laminar-turbulent flow which is statistically stationary far downstream. This conclusion is based on Fig. 1, which shows the dependence of the intermittency factor $\overline{\gamma}$ on Reynolds number, with $\frac{x}{d}$ ($x =$ distance from pipe entrance) as parameter. The method for measuring $\overline{\gamma}$ is based on the difference in mean momentum for laminar and turbulent motion when the mass flow is held fixed. If the fluid is a liquid which emerges from the pipe as a free jet, the trajectory of this jet will fluctuate between two positions [1], and the liquid can readily be separated into two parts representing laminar and turbulent flow respectively. Rotta (1956) made the first quantitative measurements by this method, and the later observations by Coles (unpublished) merely extend these data to larger values of $\frac{x}{d}$. The experiments by Coles were done in a rather off-hand way as an exercise for students, and there is some uncertainty about the proper value of the viscosity and hence of the Reynolds number at any one value of $\frac{x}{d}$. However, the data definitely

1. For an interpretation of this phenomenon assuming that the pressure drop rather than the mass flux is fixed. see Prandtl and Tietjens, *Hydro-und Aeromechanik*, Band II, Springer, Berlin, 1931, pp. 31-43. It should be noted, however, that the descriptions quoted by Prandtl and Tietjens are entirely consistent with the assumption of a constant mass flow and a fluctuating momentum. In this connection, see also several brief papers by Sackmann in *Comptes Rendus* (e.g., **239**, 220-222, 1954).

establish the persistence of the mixed laminar-turbulent regime at stations up to 10,000 diameters from the pipe entrance.

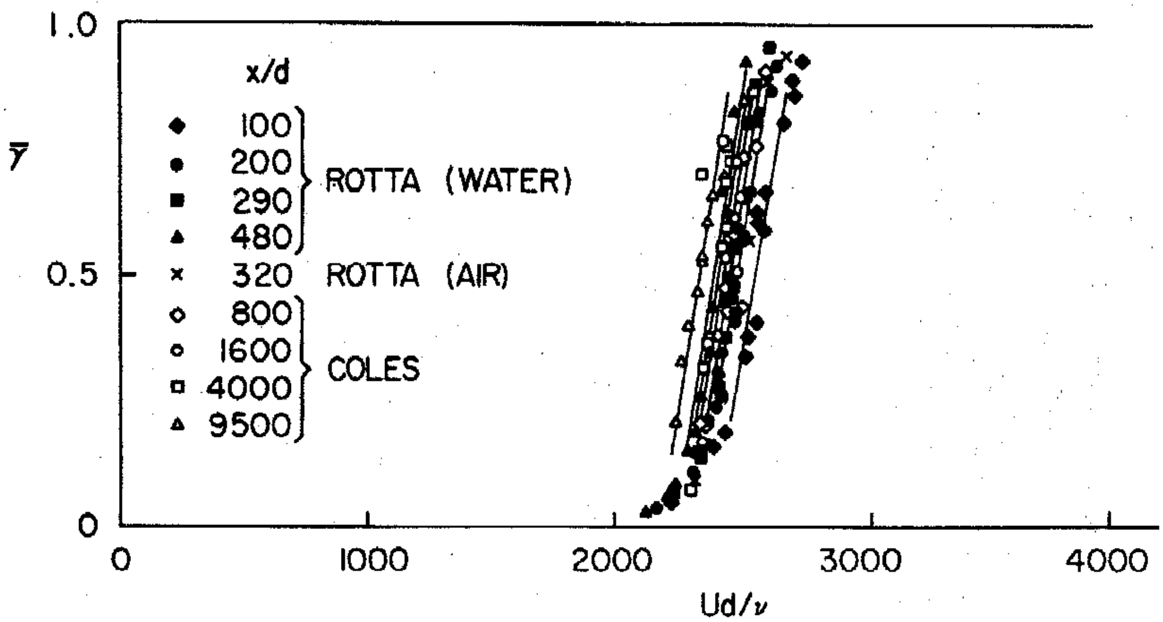


FIGURE 1
Pipe intermittency factor according to data of Rotta (1956) and Coles (unpublished) in water using the jet-momentum technique and data of Rotta (1956) in air using a hot-wire anemometer.

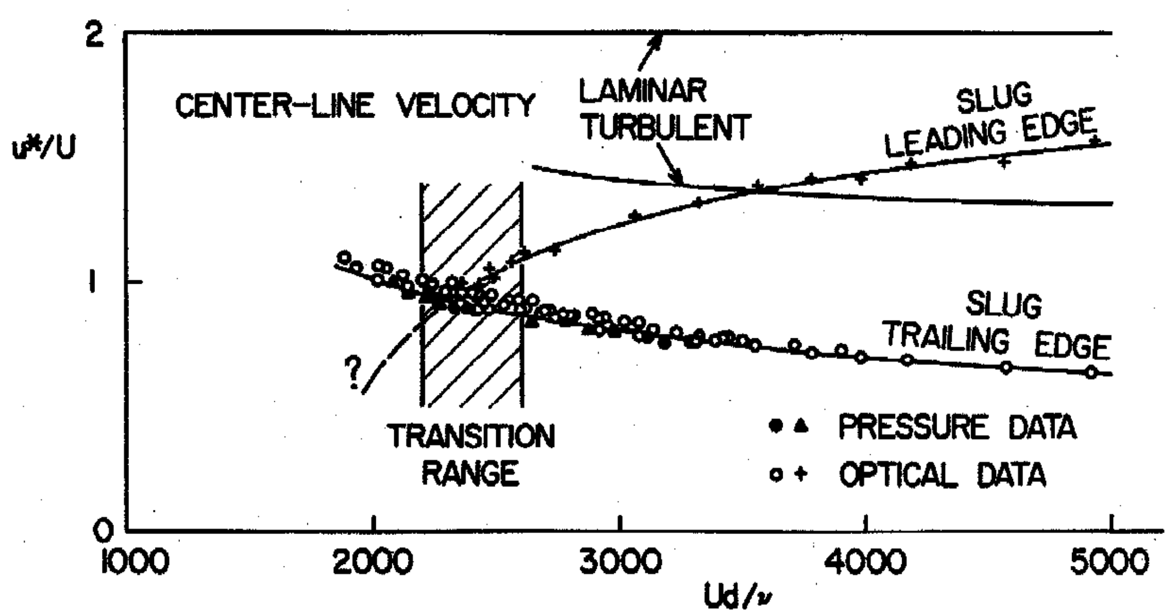


FIGURE 2
Slug velocity u^* in pipe flow according to LINDGREN (1957).
Tables 5.5, 5.8, Fig. 5. 12 (water, local pressure-drop records) and LINDGREN (1960a),
Figs 4, 5 (1.7 per cent bentonite sol, optical records).
Center-line velocity for turbulent flow from STANTON and PANNELL (1914).

Some details of pipe transition have also been described by LINDGREN (1957-1960), who used optical methods to study the flow of a birefringent liquid (bentonite sol). Much of LINDGREN's work is concerned with the evolution of individual turbulent slugs observed at successive stations relatively near the entrance, rather than with asymptotic conditions far downstream. However, one quantity which seems to be quite insensitive to the state of the flow is the slug trailing-edge velocity u^*, shown in Fig. 2. These data were obtained in various tubes with various fluids and entry conditions, using both optical and pressure instrumentation [1]. The measurements obviously extend far beyond the real transition range in both directions, so that for the most part the flows in question cannot be statistically stationary in x. LINDGREN's observations at low values of R confirm the well-known fact that turbulence originating near the entrance tends to decay far downstream; his measurements of interface velocity in this range refer to slugs which have survived long enough to be detected at two successive stations along the pipe (for documentation of these remarks see the oscillographic records in LINDGREN, 1957, Figs. 4.6, 4.7, 4.19, 4.20, as well as Table 2.5; LINDGREN, 1960*b*, Fig. 5). For the measurements at high values of R, on the other hand, a relatively low level of inlet turbulence was necessary in order to observe intermittency at all. Slugs first appeared somewhere downstream of the entrance and grew rapidly (see the oscillographic records in LINDGREN, 1957, Figs. 4.14, 5.9, 5.10; LINDGREN, 1960*b*, Figs. 6, 7). Provided that these slugs had not yet merged, LINDGREN was able to measure the slug leading-edge velocity u^* shown in Fig. 2. He does not comment on these data in terms of the implied rate of approach to an equilibrium state, and does not provide any information about flow at lower Reynolds numbers, where the relative magnitude of the leading- and trailing-edge velocities must be reversed [2].

Another phenomenon observed by LINDGREN is splitting of individual turbulent slugs (see the records in LINDGREN, 1957, Figs. 4.7, 4.19, 5.4), usually at Reynolds numbers centered in the lower transition region at about $R = 2350$. If intermittency is represented by an on-off function $\gamma(x, r, t; R)$, with $\gamma = 0$ (laminar flow) or $\gamma = 1$ (turbulent flow), the mean amplitude of this function with respect to time (and also with respect to radius) is the quantity $\bar{\gamma}(x; R)$ already plotted in Fig. 1. The mean period, expressed as the mean frequency $\bar{n}$ of turbulent slugs, can also be observed by the jet-momentum technique. For $\frac{x}{d}$ greater than about 200, ROTTA found that the dimensionless frequency $\frac{\bar{n}d}{U}$ has a maximum value of about 0.025 at a Reynolds nuber of about 2400, as shown in Fig. 3. On either side of the maximum the slug frequency decreases sharply because at lower Reynolds numbers the slug population is reduced near the entrance by the demise, and at higher Reynolds numbers by the consolidation, of individual slugs. ROTTA smoothed his original data severely (on the ground that vibration and capillary forces may have caused the jet to break up for the longer tubes), presumably in an effort

1. A slight effect of bentonite concentration was noted by LINDGREN (see 1959*c*, Fig. 4, or 1960*a*, Fig. 8). At the lowest concentrations the optical data were in good agreement with the pressure data obtained with distilled water.

2. *Note added in proof.* A limited investigation of channel flow by dye techniques under roughly corresponding conditions has been reported by G.C. SHERLIN in "Behavior of isolated disturbances superimposed on laminar flow in a rectangular pipe", *J. Res. NBS*, **64A**, 281-289, 1960.

to remove an observed increase in $\bar{n}$ with increasing $\frac{x}{d}$ at fixed R. The raw data in Fig. 3 include this effect, which may in fact be real and may represent the same splitting process described by LINDGREN.

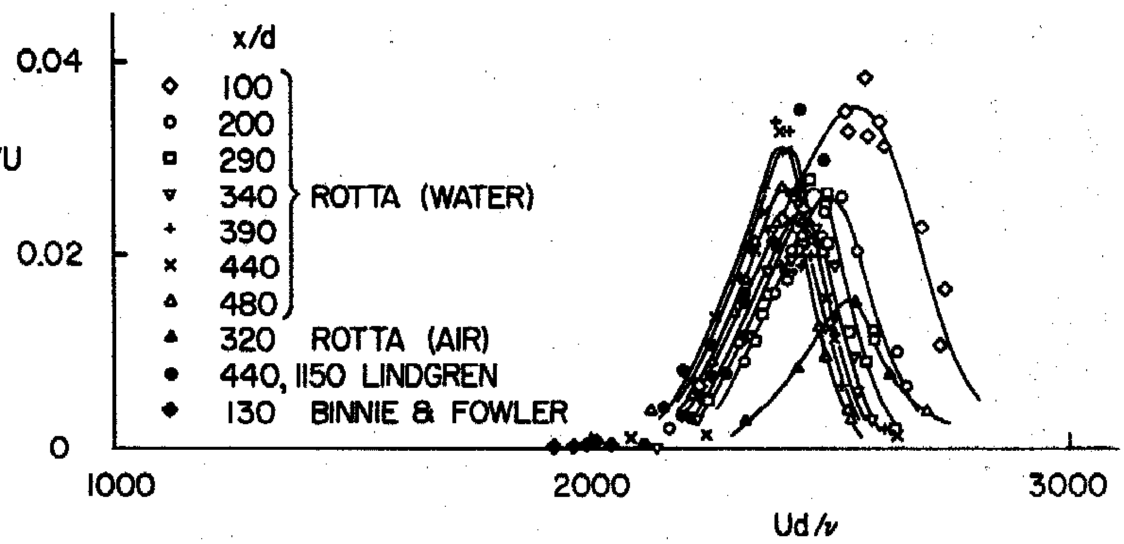


FIGURE 3

Slug frequency in pipe flow according to BINNIE and FOWLER (1947), ROTTA (1956), and LINDGREN (1957). Rotta's data obtained by jet-momentum technique in water and by hot-wire tchnique in air; remaining data obtained by optical methods in birefringent liquids.

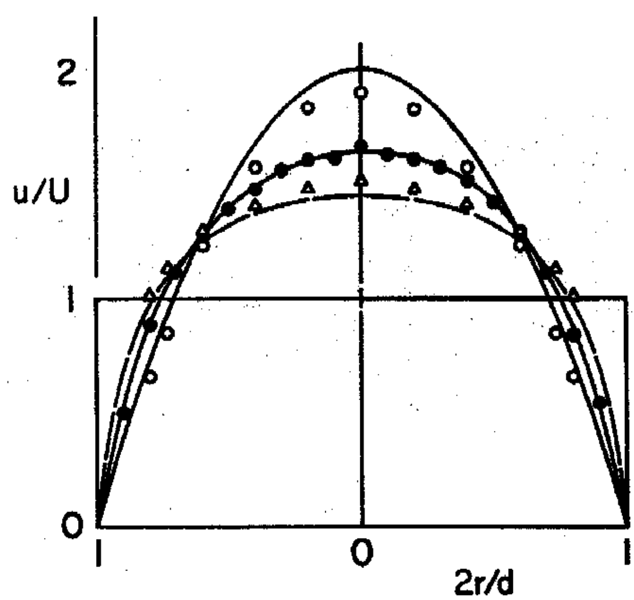


FIGURE 4

Excursion of mean-velocity profile in pipe flow at R = 2550, $\gamma = 0.7$, according to ROTTA (1956). Open circles triangles show characteristic velocities observed in laminar and turbulent regions. Solid circles show average profile measured by damped pitot tube. Solid line is parabolic laminar profile and dashed line is estimated turbulent profle, both adjusted to provide proper mass flow.

Several writers have proposed as a first approximation that intermittency in pipe flow should be viewed as a quite literal alternation between fully developed laminar and turbulent regimes. By observing the DC and low-frequency components of a hot-wire signal in a flow of air at $R = 2550$, Rotta (1956) found that the axial velocity at a fixed point fluctuated between two values characteristic of laminar and turbulent flow respectively. These values are shown in Fig. 4, together with the mean profile indicated in the same flow by a heavily damped pitot tube. Although neither of the momentary profiles yields the correct mass flow (which was independently measured by means of an upstream sonic orifice), the momentary state of the flow seems always to be close to one limiting profile or the other. Supporting evidence for this conclusion can also be drawn from Lindgren's work. The pressure records in Figs. 5.9 and 5.10 of his 1957 paper allow an estimate of the local axial pressure gradient inside a relatively long turbulent slug. At both Reynolds number studied (3180 and 3480), this pressure gradient is about 1.9 times that in the intervening laminar intervals, while the ratio of the commonly accepted friction coefficients for fully developed turbulent and laminar flow at these Reynolds numbers is about 2.0.

If the experimental indications just outlined can be applied throughout the transition Reynolds number range, some useful conclusions can be drawn about the mean flow in the vicinity of a typical turbulent slug. In a coordinate system moving with the slug velocity u^* (which according to Fig. 2 is about 0.9 U for the conditions of Rotta's experiments in air), the mean flow far downstream can be taken as steady, and an axially-symmetric stream function can be computed for the two limiting profiles in Fig. 4; i.e. for the solid and dashed lines. The resulting streamlines must have the form shown schematically in Fig. 5. Near the wall, which is now moving to the left at the velocity u^*,

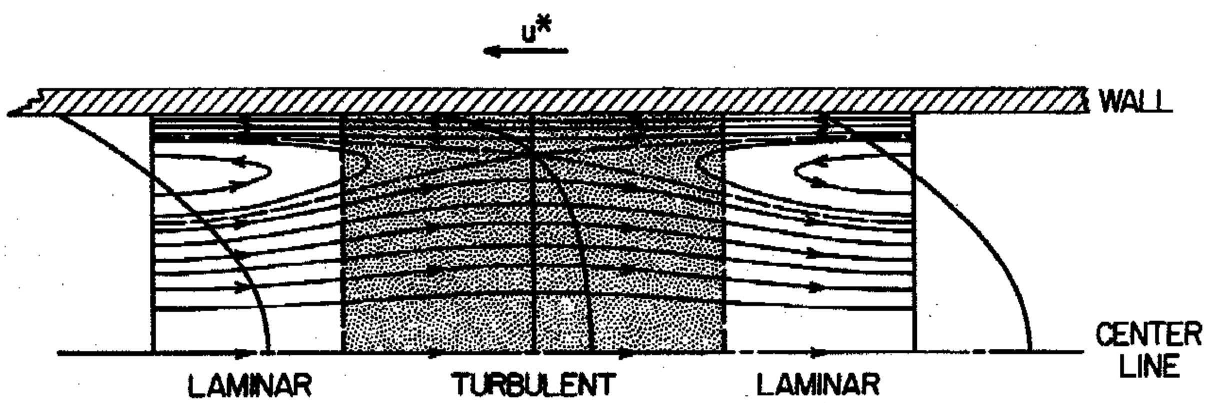
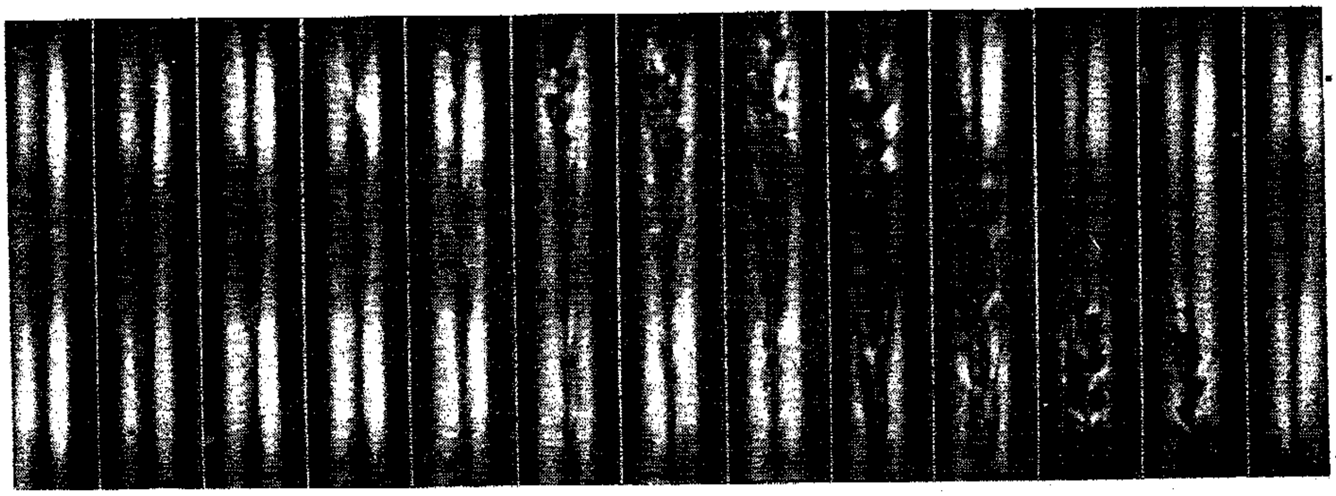
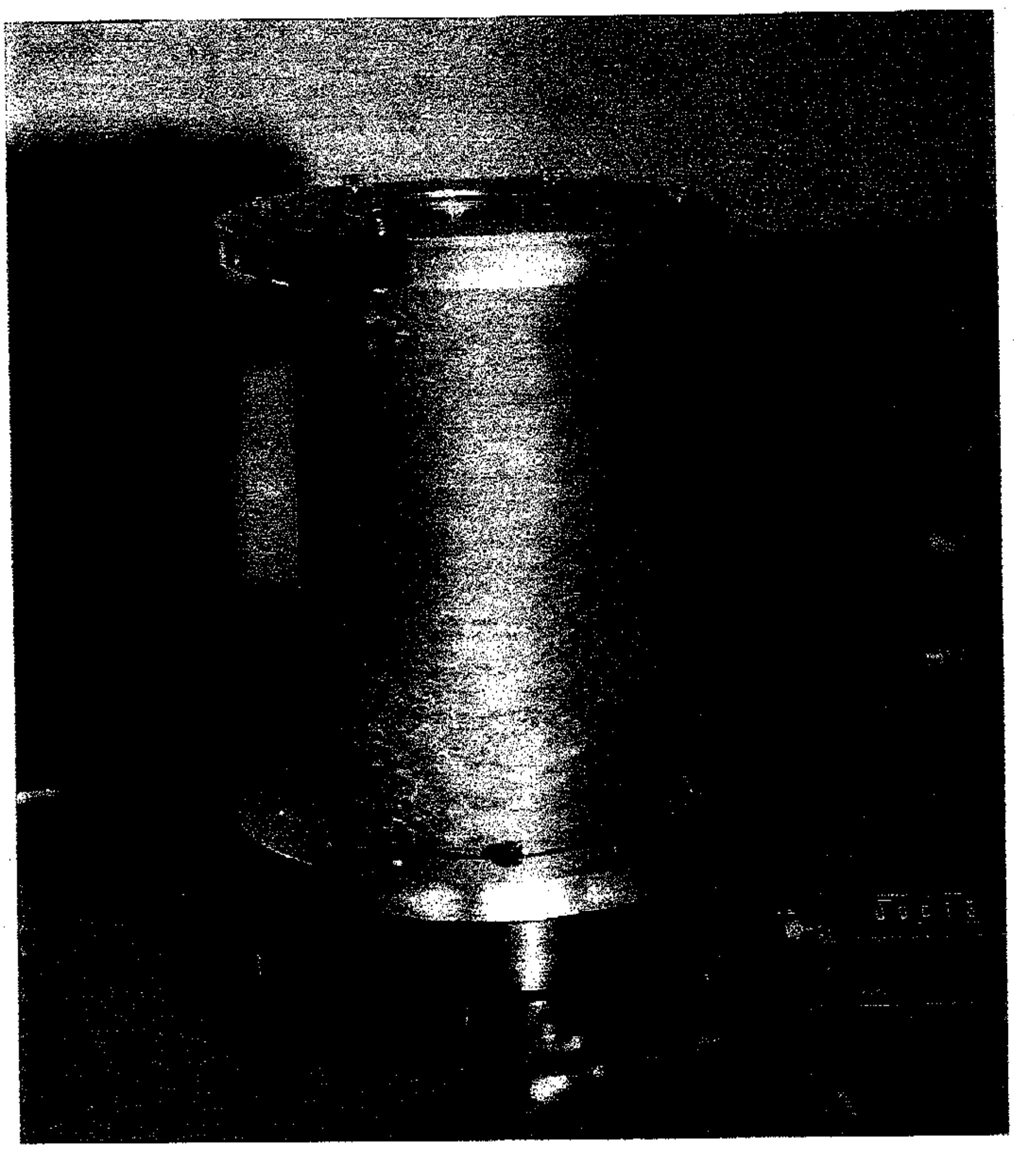


Figure 5
Schematic representation of pipe slug flow in coordinates moving with slug velocity u^*. Note reversal of flow direction near leading (right) and trailing (left) interfaces.

fluid enters the slug at the front and leaves at the rear. Near the centerline the situation is reversed; fluid enters at the rear and leaves at the front (with a relative velocity in both cases of about 1.2 u^*), meanwhile slowing down inside (to about 0.6 u^*). There is a small net flow to the right corresponding to the excess of U over u^*. The important point, however, is that a substantial fraction of the fluid entering the slug from the



6



7

Fig. 6. — Turbulent slug in pipe flow (Lindgren 1959a, Fig. 17). Flow visualization using double refraction in birefringent liquid (bentonite sol).

Fig. 7. — Spiral turbulence in circular Couette flow (Coles, unpublished). Flow visualization using suspension of aluminum powder in silicone oil.

rear reverses direction inside and emerges again at the rear, with a similar situation obtaining at the front[1].

Fig. 5, while topologically sound, has to be modified in detail to agree with existing knowledge about the size and shape of the turbulent slugs found in pipe flow. For example, the maximum slug frequency $\frac{\overline{n}d}{U} = 0.025$ from Fig. 3 and the condition $u^* = 0.9$ U from Fig. 2 imply that the slug spacing at $R = 2400$ is about 35 pipe diameters on the average. Since $\overline{\gamma}$ is about 0.4 from Fig. 1, it follows that a typical slug under these conditions is about 15 diameters long. Similar estimates based on these same jet-momentum data for other Reynolds numbers suggest that a typical slug may be of the order of 10 diameters long whenever the flow is predominantly laminar, and that a typical laminar region may be of the order of 20 diameters long whenever the flow is predominantly turbulent. Rotta also obtained results in air, as shown in his Fig. 28, which are in good qualitative agreement with these estimates.

Another feature of turbulent slugs in a real flow is a necessary asymmetry with respect to x. As shown in Fig. 6 (see also Lindgren, 1959*a*, Figs. 16 and 17), there must be a distortion of the shape of the turbulent slug, this distortion having a sense like that of the mean-velocity field. In practice the leading edge of a fully developed slug is found to be long and sharply pointed, while the trailing edge is moderately concave. Both Lindgren's flow-visualization studies and Rotta's hot-wire data indicate that the turbulence level is relatively high at the rear of the slug, and that the transition from a laminar to a turbulent state in fluid entering the slug from the rear occurs abruptly (in the case of Rotta's hot-wire measurements, in a time small compared to the time constant of his low-pass filter). The fluid leaving the slug at the front, on the other hand, undergoes a slow acceleration requiring at least several pipe diameters for completion, while at the same time the turbulence becomes progressively coarser and weaker until it is no longer detectable. Near the knee of the turbulent mean-velocity profile both transitions seem to proceed at an intermediate rate, inasmuch as the front and rear of the hot-wire signals are more nearly symmetric with respect to time at $\frac{2r}{d} = 0.8$. Although no data were obtained very near the wall, it is likely that there is again a relatively rapid change in state at the front of the slug and a slow change at the rear.

C. — Circular Couette Flow

From the point of view of transition, the circular Couette flow between concentric rotating cylinders is similar in several respects to the flow in a pipe. Although transition in Couette flow is usually understood to involve the kind of instability first studied by Taylor in his classic paper (1923), this instability actually occurs only in flows dominated by rotation of the inner cylinder. In flows dominated by rotation of the outer cylinder, on the other hand, the experimental evidence suggests that the basic motion

1. It may or may not be significant that the pattern of Fig. 5, when repeated indefinitely, resembles the cat's-eye pattern associated with the small-disturbance instability theory when the secondary motion is viewed in a coordinate system moving with the wave velocity. From this point of view the secondary motion in pipe flow is equivalent in part to a series of toroidal vortices traveling down the length of the pipe at slightly less than the mean velocity.

is stable to infinitesimal disturbances, that the flow always becomes turbulent at sufficiently high speeds, and that there is an intermediate range of Reynolds numbers in which a stable mixed laminar-turbulent configuration is observed provided that sufficiently strong disturbances are present. These are the same properties which distinguish the pipe flow, the major difference between the two flows being that any tendency toward periodicity of the intermittency phenomenon in the transition region is more rigidly enforced in the case of the Couette flow by the closed geometry.

The typical configuration in the intermittent regime of Couette flow is a spiral band of turbulence, as shown in Fig. 7. This spiral rotates at approximately the mean angular velocity of the two cylinders, without changing its shape or losing its identity, and observations of the flow from either wall reveal a remarkably regular alternation of laminar and turbulent motion. For one experimental apparatus, the range of operation within which this spiral turbulent pattern occurs is shown in Fig. 8. The base of the figure is made up of the two Reynolds numbers $R_i = \frac{\omega_i r_i^2}{\nu} = \frac{U_i r_i}{\nu}$ and $R_0 = \frac{\omega_0 r_0^2}{\nu} = \frac{U_0 r_0}{\nu}$ for the inner and outer cylinders (r = radius, ω = angular velocity, U = sur-

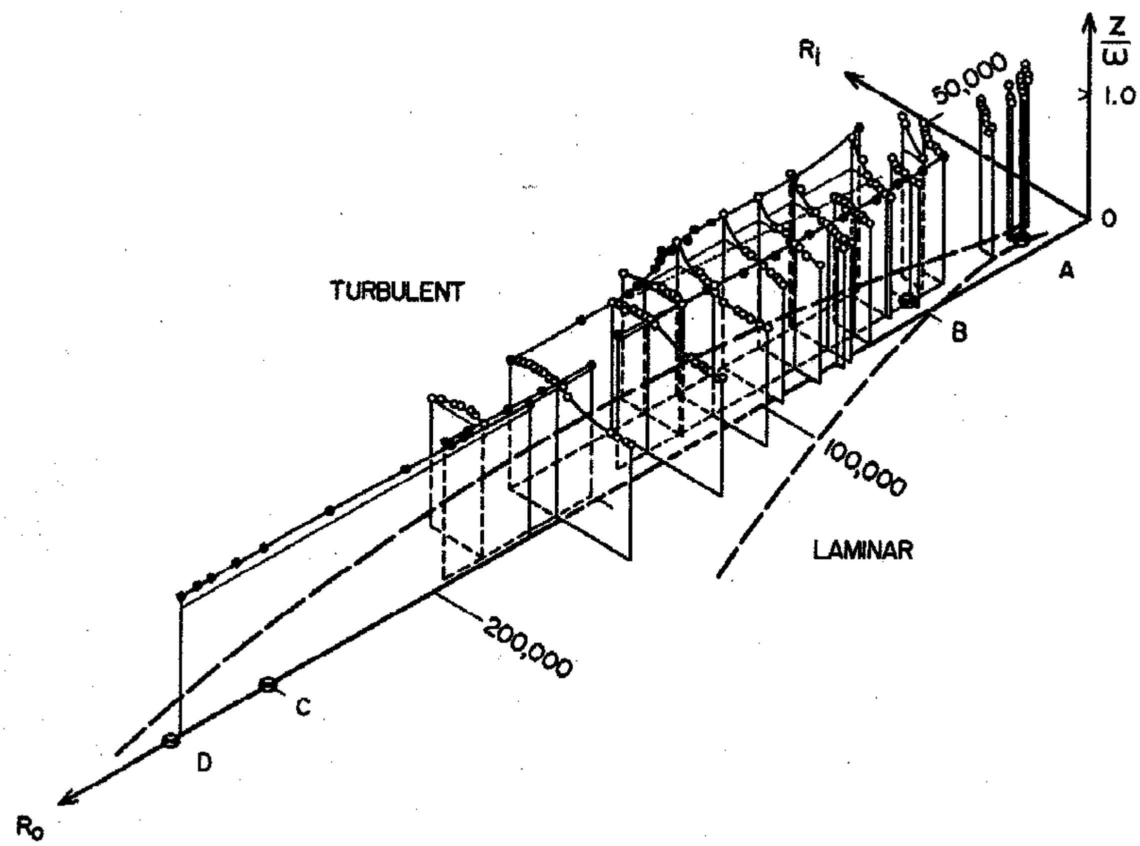


FIGURE 8
Angular velocity ω^* of spiral turbulence pattern in circular Couette flow as function of inner and outer cylinder Reynolds numbers according to Van Atta (unpublished). Open circles are data for constant R_0; solid circles are data for constant R_i. Dotted line is Taylor boundary. Point « A » corresponds to photograph in Fig. 7; point « B » to data of Fig. 9; point « C » to data of Fig. 10; point « D » to observations of second mode (two spirals).

face velocity), and the ordinate is the angular velocity of the turbulent pattern, ω^*, normalized with respect to the mean angular velocity of the two cylinders, $\frac{(\omega_0 + \omega_i)}{2}$ (compare Fig. 2 for the slug velocity in pipe flow). For most of the figure the cylinders are rotating in opposite directions, with the speed of the inner cylinder small compared to the speed of the outer one. The observations in Fig. 8 refer to flow of air in a machine with an annular working space having an outside diameter of about 90 cm, a length of about 150 cm, and a thickness of about 5 cm. The ends of this space are closed by plates which rotate with the outer cylinder but which can be moved axially (while the cylinders are running) to change the length of the working space between the limits 80 and 140 cm. The instrumentation consists at present of several hot-wire anemometers. For the measurements reported in Fig. 8, the data near the center of the transition region were obtained by observing the flow at a radius midway between the two cylinders. The intermittency factor $\overline{\gamma}$ at this radius was usually between 0.3 and 0.7 but was not measured accurately, inasmuch as the primary purpose of these measurements was to find an operating condition for which the mixed flow was particularly regular and stable and thus suitable for more elaborate experiments. An increase in speed within the transition region usually caused the turbulence to spread throughout the flow at mid-radius, although the spiral configuration could still be detected by observations made close to one wall or the other (an example in which the inner cylinder is at rest is given in Fig. 10 below). A decrease in speed was usually accompaneid by increasing randomness and by degeneration of the spiral structure until the turbulence finally died out completely. Following this transition to fully laminar flow, it was usually necessary to increase the speed appreciably or to introduce an artificial disturbance (such as an air jet at one wall) before the turbulence could be reestablished. Under appropriate conditions the secondary motion associated with the Taylor instability sometimes served as the necessary finite disturbance for tripping the flow. For laminar motion this particular instability is associated with the dotted line in Fig. 8, and there is some indication that a related instability may play a role in the spiral turbulence, even though the latter phenomenon when present has a preemptive control over the nature of the motion. Except for the hysteresis just mentioned, which means that two distinct states (one completely laminar, the other partially turbulent) could be observed at some speeds, the spiral flows in the transition region appeared to be unique for a given geometry regardless of the way in which they were established.

The photograph in Fig. 7, corresponding to the point marked « A » in Fig. 8, was obtained using a smaller but geometrically similar apparatus equipped with glass cylinders and filled with a silicone oil. The flow-visualization technique was essentially the same as that described by Schultz-Grunow and Hein (1956). Visual observations with the small apparatus, as well as hot-wire data obtained in the large one, suggest that the helix angle of the spiral turbulence pattern is usually close to 60° (measured from the axis of the cylinders). The ordinary configuration is a single spiral, but at least one case has been found, at the point marked « D » in Fig. 8, of a second mode involving two spirals 180° apart. Both of these observations are subject to later revision, inasmuch as the data so far obtained refer only to a gap/inner radius ratio of about 1/8.

At two operating conditions, marked « B » and « C » in Fig. 8, the cross-sectional shape of the spiral turbulent region is known from intermittency measurements carried

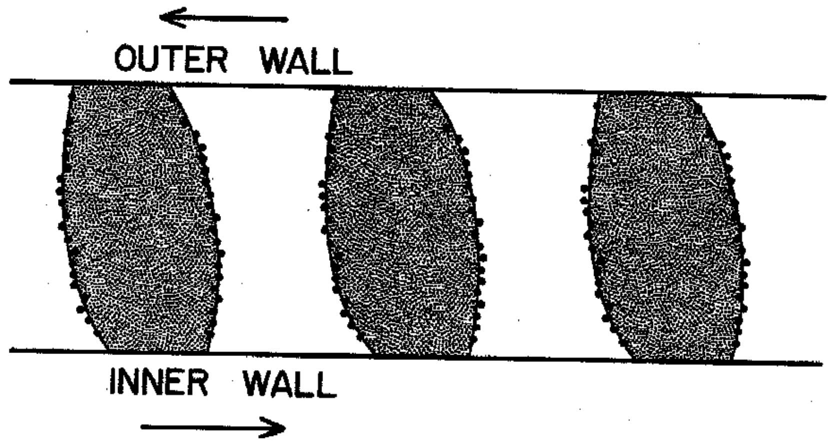


FIGURE 9
Interface geometry in circular Couette flow according to Van Atta (unpublished). Operation at point « B » in Fig. 8; opposite rotation with $R_o = 50{,}000$ and $R_i = 5{,}600$.

out by C. Van ATTA and H. OGURO respectively at GALCIT. The data are shown in Figs. 9 and 10. In both cases the intermittency factor was first measured as a function of radius. For the condition « C », the tangential position of the laminar-turbulent interfaces was then estimated with the aid of multiple oscillograph records. For the condition « B », Van ATTA developed a method of adding two hot-wire signals obtained at different radii and measuring the intermittency factor of the composite signal. These data gave directly the mean tangential position of one interface at various radii with respect to the position of the other interface near one wall.

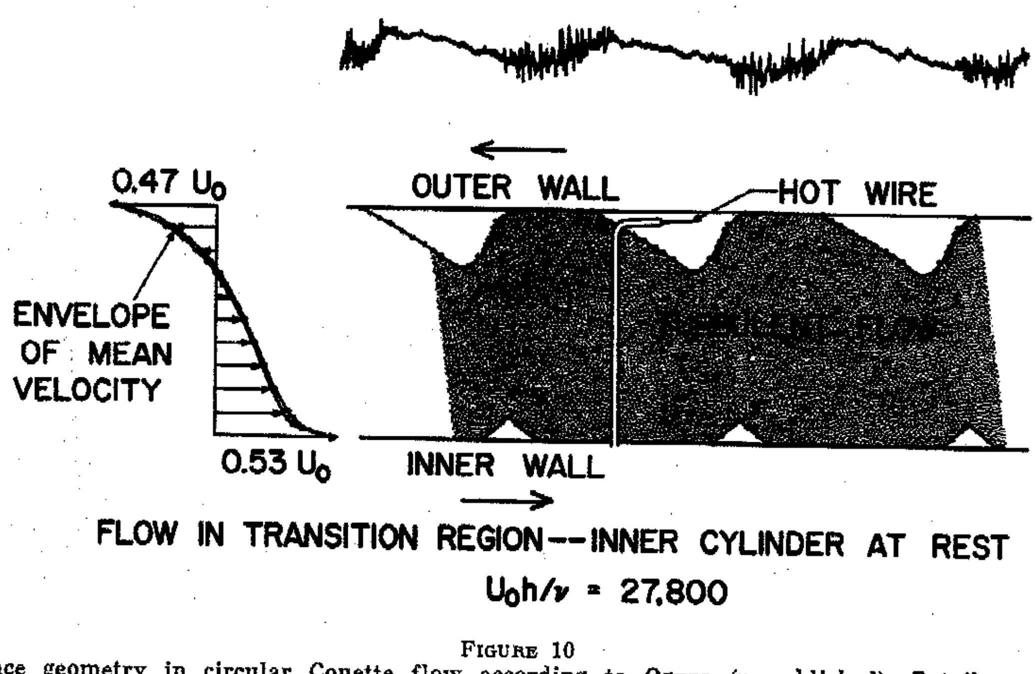


FIGURE 10
Interface geometry in circular Couette flow according to Oguro (unpublished). Detail at left is envelope of tangential mean-velocity profile. Operation at point « C » in Fig. 8; inner cylinder at rest with $R_o = 250{,}000$.

The measurements in circular Couette flow clearly demonstrate the same phenomenon already pointed out in connection with pipe flow. In the mixed laminar-turbulent configuration, fluid is again observed to pass from a laminar region into a turbulent one and out again. The example in Fig. 10 shows that when the overall intermittency level is not much less than unity the tangential velocity profile has a characteristic turbulent shape throughout the flow. The profile corresponding to Fig. 9 has not yet been measured, but it is expected that the variations in mean velocity at a fixed radius will be relatively larger in this case because of the greater extent of the laminar region.

D. — Boundary-layer Flow

The main features of transition so far described are repeated, with some important differences, in boundary-layer flow. The transition region is marked by the passage of turbulent spots which originate at some point upstream and grow in size as they are carried downstream by the ambient flow. These spots have a characteristic shape, a characteristic velocity, and a characteristic growth rate, all of which depend only slightly if at all on Reynolds number.

The concept of random turbulent spots is originally due to EMMONS (1951), who assumed on the basis of experiments with a water table that the spots could be treated as independent of each other and that the growth rate could be taken as constant. EMMONS also introduced a source density function $g(x, z, t)$ to describe the spot production process, and showed that the intermittency factor $\overline{\gamma}$ for statistically stationary transition could be written as

$$\overline{\gamma}(x, z) = 1 - e^{-\int_D g(\xi, \zeta, \tau)\, d\xi\, d\zeta\, d\tau}$$

where x and z are coordinates parallel to the surface and D is the upstream domain

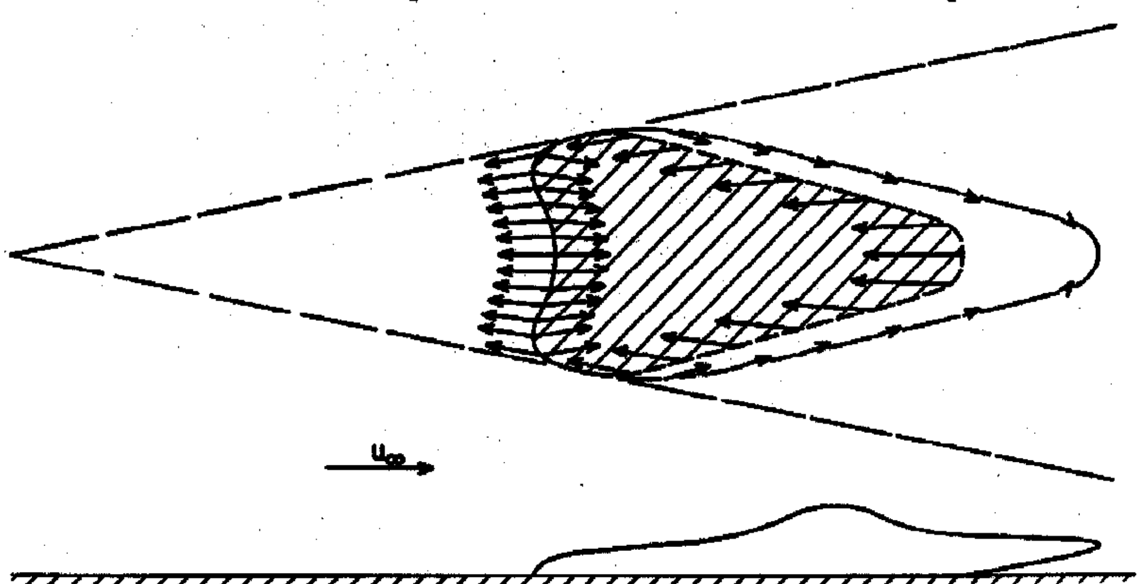


FIGURE 11
Geometry of artificial turbulent spot in boundary-layer flow according to SCHUBAUER and KLEBANOFF (1955). Arrows show estimated direction and relative magnitude of interface velocity with respect to fluid near wall and near free stream.

of influence, here assumed conical in space and time, of the point (x, z, t). EMMONS' assumption of a constant growth rate has since been verified by detailed measurements of spot growth in a boundary layer by SCHUBAUER and KLEBANOFF (1955). The technique eventually adopted by these authors, following MITCHNER (1954), was to generate artificial spots by means of an electric spark. The shape of the spot as inferred from hot-wire traces is shown in Fig. 11. The comical property of the spot geometry seems to be well established, at least for the planform. It follows that the instantaneous velocity of propagation of the boundary is in the direction of the radius vector through the point of spot origin, and is proportional to the magnitude of this vector. Alternatively, the spot boundary can be taken as stationary in coordinates $\frac{x}{u_\infty t}, \frac{z}{u_\infty t}$ (but not necessarily $\frac{y}{u_\infty t}$), where u_∞ is the free-stream velocity. SCHUBAUER and KLEBANOFF found the interface velocity (with respect to the wall) to be very nearly $0.5\ u_\infty$ at the trailing edge and $1.0\ u_\infty$ at the leading edge, and this information in turn allows a rough estimate to be made of the local fluid velocity with respect to the spot boundary, also shown in Fig. 11. Near the wall, where the fluid is almost at rest, the relative flow is into the spot at the front and out at the rear. Near the free stream there is the expected flow into the spot at the rear, but the leading interface is almost stationary in the fluid and is not all active in the entrainment process associated with the growth of the spot.

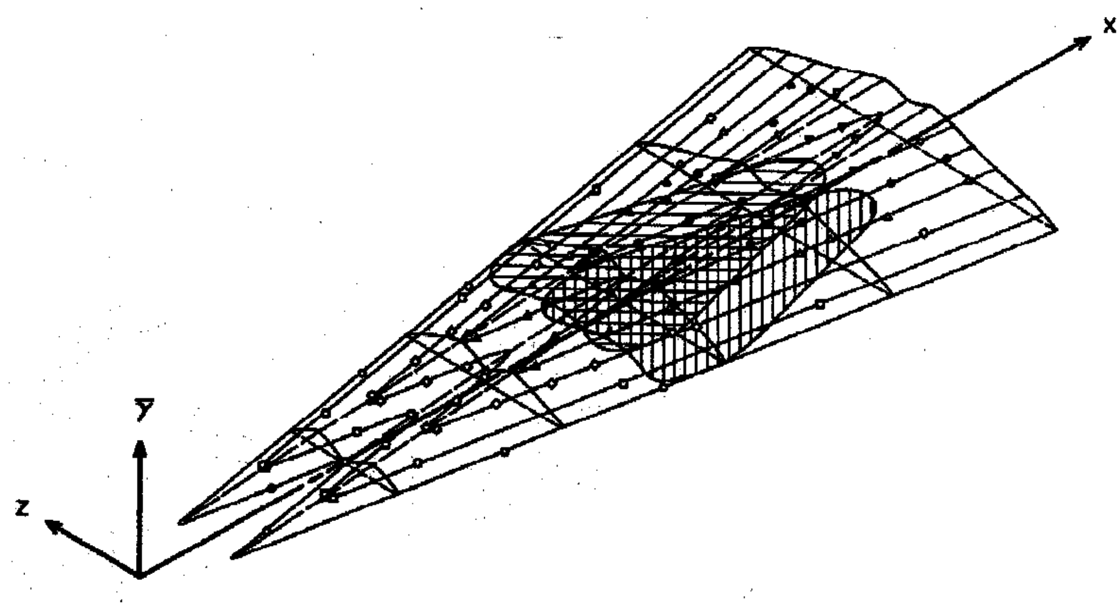


FIGURE 12
Relative intermittency factor for a pair of artificial spots in boundary-layer flow according to ELDER (1960). Spot geometry estimated on basis of Fig. 11.

EMMONS' other major assumption, that turbulent spots behave independently of each other, has been closely verified by ELDER (1960). Fig. 12 shows the observed interaction, or better the absence of interaction, for two artificial spots generated side by side and simultaneously at regular intervals by the spark technique in air. ELDER measured the intermittency factor $\overline{\gamma}$, here defined as the duration of turbulent flow compared to the time interval between sparks, as the pair of spots traveled downstream in a boundary layer. Although ELDER does not report the times of spot arrival and departure separately, so that no direct inferences can be drawn about the shape of the spots in question, the observations in Fig. 12 are seen to be reasonably consistent with the spot geometry as determined by SCHUBAUER and KLEBANOFF. ELDER also used a dye technique in water to observe the shape of a boundary-layer spot directly, with the notable result shown in Fig. 13.

Finally, DHAWAN and NARASIMHA (1958) have studied the intermittency factor $\overline{\gamma}(x)$ experimentally for transition in a flat-plate boundary layer under a wide variety of conditions. They concluded on the basis of their own and other data that EMMONS' source density function $g(x, z, t)$ could best be represented by a delta function in x; in other words, $g = g_0 \, \delta(x_t)$, where g_0 independent of z and t. If the turbulent spots originate along the line $x = x_t$, rather than uniformly in x as assumed by EMMONS, the intermittency factor for $x - x_t > 0$ can be written

$$\overline{\gamma}(x) = 1 - e^{-c\frac{(x-x_t)2}{\overline{x}^2}}$$

where c is a geometrical factor of order 2/3 (the ratio of the projected spot area to the area of the circumscribed rectangle) and the quantity $\overline{x}$ is a characteristic length for the transition region (defined here as the average distance required for the front of one spot to overtake the rear of another when the average period between successive spots is computed by taking the spot width at $x = \overline{x}$ as the active length for the line source of strength g_0 at $x = x_t$). DHAWAN and NARASIMHA also emphasized in their paper the principle of superposition, or alternation of properties, as a means of representing flow in a transition region. By weighting the turbulent and laminar contributions by factors $\overline{\gamma}$ and $1 - \overline{\gamma}$ respectively, they were able to fit not only the surface friction distribution but even the details of the mean-velocity profile. This procedure assumes, however, that the properties of turbulent boundary-layer flow are know at arbitrarily low Reynolds numbers, either through extrapolation or through direct measurement. Such an assumption seems to be essentially unprovable, unless the properties in question can be associated with spot flow *per se* or unless some quantity can be found (corresponding to the mass flow in a pipe) which is common both to flow in a spot and to flow in a fully turbulent region.

For boundary-layer flow at large Reynolds numbers, the problem of intermittent transition is replaced by the problem of intermittent turbulence at the irregular free boundary of the turbulent flow. The case of a rough wall has been studied by CORRSIN and KISTLER (1954), and definitive measurements of flow structure for the case of a smooth wall have been made by KLEBANOFF (1954). KLEBANOFF's measurements of mean-velocity profile $u(y)$ and intermittency factor $\overline{\gamma}(y)$, shown in Fig. 14, will be discussed further after a brief description of intermittency measurements in other free shear flows, all of which differ from the flows so far described in the important respect that interfaces are propagating into regions originally free of vorticity.

E. — Free Shear Flow

In Fig. 14 are shown mean-velocity profiles characteristic of the plane wake, the circular jet, and the mixing layer, as observed by TOWNSEND (1949), CORRSIN and KISTLER (1954), and LIEPMANN and LAUFER (1947) respectively. Also shown are inter-

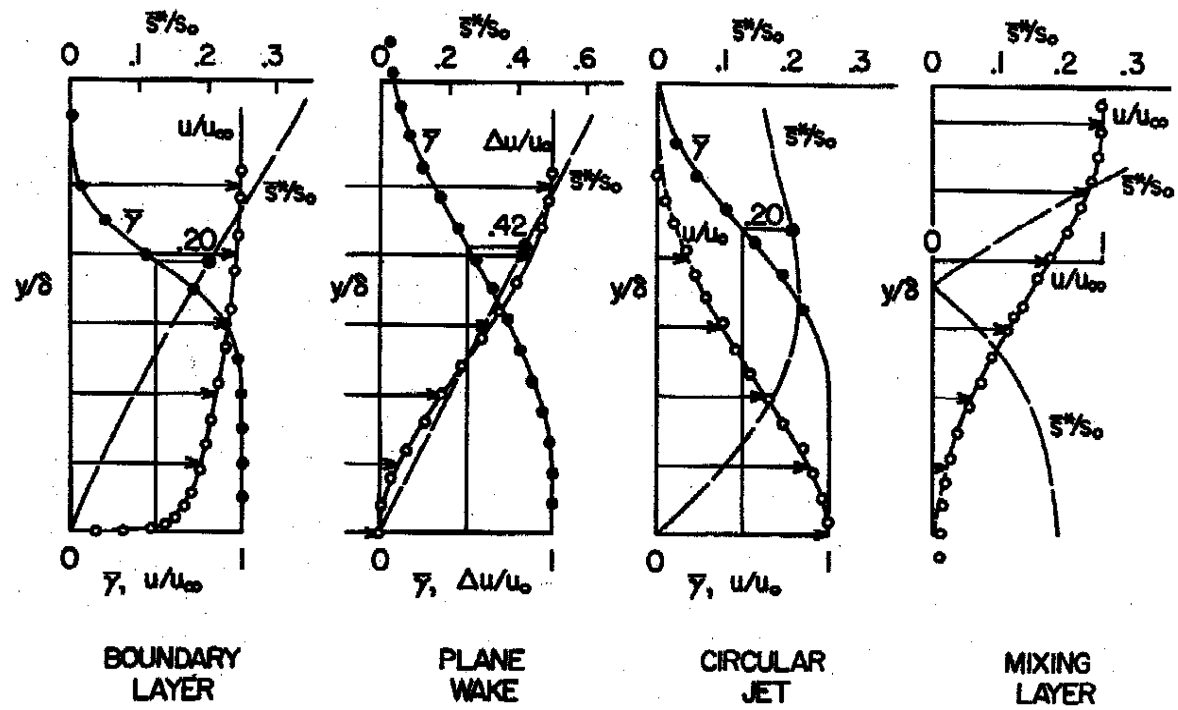
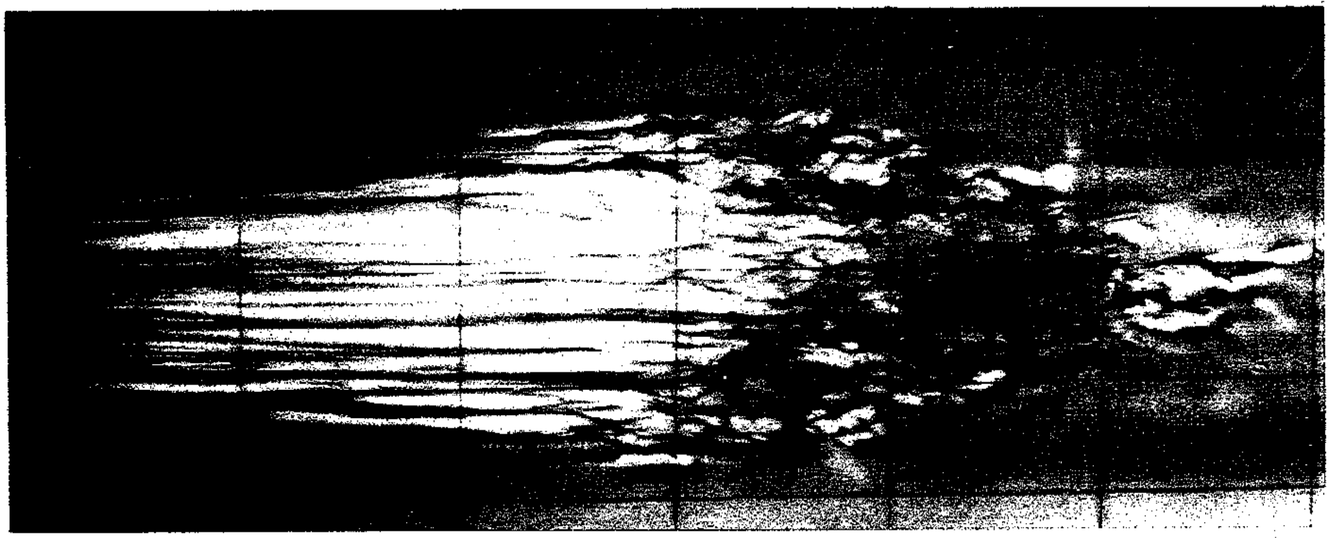


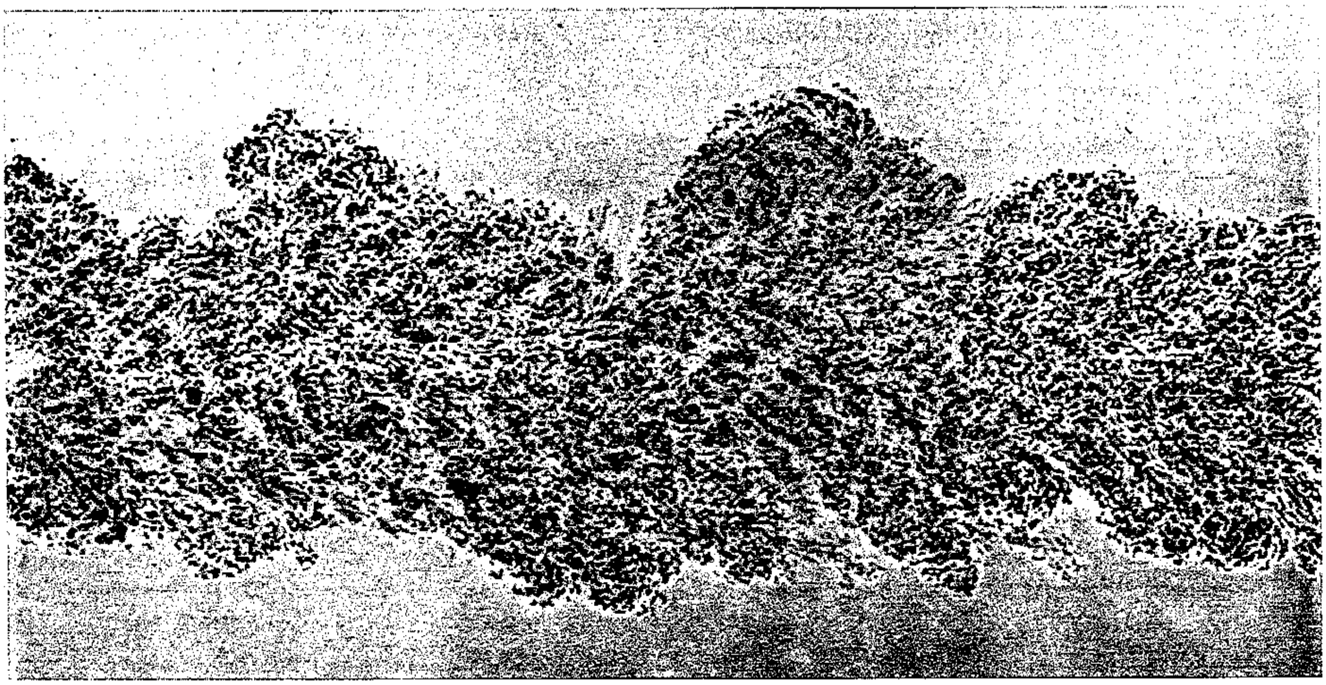
FIGURE 14
Mean velocity, intermittency factor, and interface velocity in free shear flow according to KLEBANOFF (1954), TOWNSEND (1949), CORRSIN and KISTLER (1954), and LIEPMANN and LAUFER (1947). See text for definitions of s^* and s_0.

mittency-factor distributions for the first two of these three flows. In all cases the data exhibit the appropriate similarity in terms of a suitable characteristic velocity U, defined below, and characteristic length δ, defined as twice the distance in which half the mean-velocity change occurs for the jet, the wake, and the wake component of the boundary layer. The intermittency properties, including the mean position of the laminar-turbulent interface and the standard deviation of the (essentially Gaussian) probability density for fluctuations about the mean, also show a close similarity in terms of $\frac{y}{\delta}$. Fig. 14 indicates that the fully turbulent core flow is relatively smaller for the plane wake than for the jet or boundary layer; the case of the circular wake and the plane jet have not been studied experimentally in these terms, although the former case can be represented qualitatively by the photograph shown in Fig. 15.

Because of the random nature of the intermittency signals which are typical of free shear flows, it is extremely difficult to measure the local propagation velocity of an interface with respect to the fluid. A discussion of the possible occurrence of zero or



13



15

Fig. 13. — Turbulent spot in boundary-layer flow (Elder, 1960, Fig. 2). Flow visualization in water using up-stream dye injection at surface.

Fig. 15. — Turbulent axially symmetric wake (Aberdeen Proving Ground). Flow visualization in firing range using shadowgraph observation of density fluctuations in air.

negative velocities is therefore somewhat academic, although the example of spot behavior in boundary layers suggests that this possibility should not be dismissed out of hand. Only one public attempt has been made, by CORRSIN and KISTLER (1954), to estimate the local interface velocity s^*. They proposed a formula $s^{*2} \sim \nu \; \overline{\omega' \omega'}$ (where $\omega' =$ vorticity fluctuation) which is based in part on a model of an oscillating Rayleigh flow and in part on dimensional reasoning. This formula prohibits negative values of s^* but is otherwise unexceptionable, especially if s^* is large and positive. On the other hand, the mean entrainment velocity $\overline{s^*}$ is known experimentally to be independent of viscosity in free shear flows with similarity. If it is assumed that the entrainment of non-turbulent fluid is determined by energy processes which are associated with the largest eddies in the flow, it is natural to consider (among other alternatives) a characteristic velocity s_0 defined in terms of turbulence production [1] by the formula

$$\rho\, s_0^3\, S = \int \Phi_t \, dV$$

As applied to fully developed shear flows, the volume V in this formula includes the whole lateral extent of the shear flow over unit length in the flow direction; the surface S is the part of the bounding surface made up of laminar-turbulent interfaces; and Φ_t is the local turbulence production, defined in Cartesian coordinates as the double sum $-\rho\, \overline{u_i' u_j'} \dfrac{\partial \overline{u_i}}{\partial x_j}$. The form of this definition for s_0 is controlled largely by the similarity laws for Φ_t and by the necessity for considering both plane and axially-symmetric flows. For the plane wake, for example, the formula reduces within the boundary-layer approximation to

$$\rho\, s_0^3 = \int_{-\infty}^{\infty} \tau \frac{\partial \overline{u}}{\partial y}\, dy$$

Simple numerical calculations using the experimental data of Fig. 14 then show that the ratio $\dfrac{s_0}{U}$ is about 0.19 for the mixing layer (U = free stream velocity u_∞), 0.18 for the circular jet (U = velocity u_0 on centerline), 0.25 for the plane wake (U = velocity defect u_0 on centerline), and 0.04 for the boundary layer. In the latter case the velocity gradient $\dfrac{\partial \overline{u}}{\partial y}$ for the turbulence production has been evaluated for the wake component alone, and the characteristic velocity U has been taken as the velocity defect at the wall in the equivalent wake (see COLES, 1956).

For each of these same flows it is a simple matter to compute the mean velocity normal to any surface $\dfrac{y}{\delta} =$ constant. When made dimensionless in the appropriate way this normal velocity will depend only on $\dfrac{y}{\delta}$ except for a slight effect of viscosity in the

1. These considerations are of an extremely preliminary nature, and are not intended to demonstrate a general principle for the problem of interface propagation. They are certainly influenced, perhaps favorably and perhaps not, by the experimental fact that negative velocities do sometimes occur at interfaces propagating into turbulent regions, as pointed out earlier in connection with transition.

case of the boundary layer. Finally, the mean entrainment velocity $\frac{\overline{s^*}}{U}$ can be arbitrarily defined as the value of the dimensionless normal velocity at the point where $\overline{\gamma} = \frac{1}{2}$, assuming that the latter is known. For the boundary layer and the circular jet the ratio of the calculated mean entrainment velocity $\overline{s^*}$ to the characteristic production velocity s_0 is about 0.20, and quite comparable values would evidently be obtained for both sides of the mixing layer, according to Fig. 14, for any reasonable estimate of the intermittency factor distribution. The ratio $\frac{\overline{s^*}}{s_0}$ for the plane wake, unfortunately, is about twice as large.

F. — Discussion

Among several related questions raised by this study of interfaces and intermittency, one of the most important concerns the remarkable stability of the mixed flows already described. It seems that nature does not ordinarily provide a continuous range of states varying from fully laminar to fully turbulent flow [1]. If both types of flow are present they are distinct, in the same sense that the liquid and gaseous states are distinct for any ordinary fluid. At least in the case of transition, the turbulent regions have a characteristic geometry and a characteristic propagation velocity which are so regular that a definite mechanism must be involved. It may therefore be constructive to approach this problem of transition from above rather than from below, by supposing that it is really turbulent flow which is the normal state, while laminar flow is abnormal. The stability of the turbulent state can then be investigated in various ways, say by paraphrasing the usual comparison of the rates of production and dissipation of fluctuating energy with emphasis on the viability of turbulent motion at low Reynolds numbers. The important property common to the transition flows being discussed here then becomes the existence of interfaces propagating abnormally; i. e., from laminar into turbulent regions, so that fluid passing through these interfaces is passing from the turbulent state to the non-turbulent one.

By definition, the term " turbulence " implies the existence of random non-steady three-dimensional vorticity. In two-dimensional incompressible flow, vorticity is generated at solid boundaries and remains permanently attached to individual fluid elements except for the diffusive effects of viscosity. In three-dimensional flow, however, the generation and diffusion of vorticity are subordinate to a phenomenon which has been variously described as vortex stretching or as a kind of gyroscopic precession in the fluid. This latter phenomenon is non-dissipative and hence in principle reversible, and may be an important factor in the processes which control the appearance, flow, and disappearance of turbulent energy.

The paper by Rotta (1956) includes an attempt to estimate the reversible part of the energy conversion between mean flow and turbulence in the vicinity of an interface in

1. Exceptions to this rule are of course known. For example, the cellular instabilities which occur in circular Couette flow and in free convection between parallel planes are the first stage of a monotonic and essentially reversible transition from a discrete to a continuous spectrum for deviations from mean flow quantities.

pipe flow. However, this estimate is not entirely satisfactory, as it assumes that $\frac{\partial p}{\partial x}$ is independent of radius but that $\frac{\partial \tau}{\partial r}$ is not. The effect of radial pressure gradients is therefore not taken into account. The actual magnitude of these pressure gradients in pipe flow (or of the corresponding gradients in any other intermittent flow) is so far unknown, but the observed reversal in flow direction relative to the interfaces, shown in Fig. 5, suggests that they may be important. It seems to be typical of intermittent flow that the relative mean velocity decreases during a laminar-turbulent transition and increases during a turbulent-laminar one. These changes in mean velocity are consistent with a relatively sudden application or removal of turbulent stresses, which tend to make the mean flow more nearly uniform, but are also consistent with a process of kinetic-energy conversion between mean and random flow fields. In the case of an irregular interface like that characterizing the boundary layer, it can be conjectured from a knowledge of spot behavior that the rearward-facing interfaces are most active in the entrainment process. Consequently it is entirely plausible that non-turbulent fluid moving at free-stream velocity can overtake these slower-moving interfaces from the rear.

Several other transition problems also involve the effect of intermittent turbulence on its environment and vice versa. One such problem is the calming effect first pointed out for boundary-layer flow by Schubauer and Klebanoff (1955). At the rear of a turbulent spot the surface friction is left abnormally high as the fluctuations disappear. The velocity profile is left correspondingly full, and reverts only slowly to the normal (Blasius) profile, meanwhile showing an increased stability which prevents any new transition immediately behind the spot unless quite large disturbances are present. A similar situation may occur in pipe flow, if it is true that the laminar intervals between turbulent slugs have a characteristic minimum length of about 20 diameters. A related problem in pipe flow is the splitting process described by Lindgren (1957). In a pipe flow in which the intermittency factor is approaching an equilibrium value from below, there is apparently a strong tendency for turbulent slugs to grow to a definite length and then to divide rather than to continue growing. Thus a single slug produced artificially in an otherwise quiet pipe flow (at $R = 2350$, say) might be expected to show repeated splitting. Lindgren's data also produce the impression that it is the leading edge of a slug which is the active region for this splitting process and for growth in general.

A preliminary contribution to the subject of energy processes in mixed flows has recently been made by Laufer (unpublished), who studied the decay of fully-developed pipe turbulence following a gradual transition from one cylindrical pipe to another of larger diameter[1]. In this particular experiment the Reynolds number decreased from 3450 to 1380 as a result of the increase in area. The decay of the turbulence was found to be free of intermittency arising through the discharge of individual slugs (e. g., Laufer's measurements of r. m. s. velocity fluctuation do not show the characteristic

1. *Note added in proof*. Similar experiments in pipe flow have recently been reported by M. Sibulkin *in* " Transition from turbulent to laminar pipe flow ", *Convair Sci. Res. Lab. Research note* 52, Oct. 1961.

minimum away from the centerline described by Rotta), and the flow in question can be roughly visualized as a statistically stationary tongue of decaying turbulence of vaguely conical shape (cf. Figs. 6 and 7 of Lindgren, 1959*a*).

Laufer's most arresting result concerns the virtually complete similarity of the spectrum for the axial velocity fluctuations on the pipe centerline during the decay of the turbulence. As shown in Fig. 16, this spectrum is closely exponential in form,

$$E = \rho \frac{\overline{u'u'}}{2} = E_0 \int_0^\infty \frac{\overline{u}F}{d}\, d\frac{df}{\overline{u}} = 1.36\, E_0 \int_0^\infty e^{-\alpha\frac{af}{\overline{u}}}\, d\frac{df}{\overline{u}}$$

(where f = frequency, F = spectral density, $\overline{u}$ = mean velocity on centerline, d = diameter, and E_0 = reference energy per unit volume). The numerical coefficient 1.36 was determined experimentally from the measured spectrum at the first station, where $E = E_0$ by definition. The parameter α ($4L/d$ in Laufer's notation) is evidently given by

$$\alpha = 1.36 \frac{E_0}{E}$$

and depends on x. In fact, this dependence was also found experimentally to be of exponential form,

$$\alpha = A\, e^{\frac{ax}{d}}$$

These measurements show a strange kind of equilibrium in which the energy at zero frequency is essentially unaffected by the decay. The integral scale[1] for the spectrum in Fig. 16 is very nearly αd, and increases rapidly with x. The microscale, defined as usual in terms of the second moment of the measured spectrum in Fig. 16 or in terms of the second derivative of the autocorrelation, is smaller by a factor 2π. After 40 diameters of decay, the energy on the centerline has decreased by a factor of 10, and both scales exceed the pipe diameter.

On the basis of these and other measurements, Laufer suggests the possibility that viscous dissipation alone may not be sufficient to account for the disappearance of turbulence in this particular flow. The energy balance on the centerline for the axial component of the turbulence can be written

$$\begin{array}{ccccccc} -140 & & -65 & & +2530 & & -2605 \\ \overline{u}\frac{\partial}{\partial x}\left(\frac{\overline{u'u'}}{2}\right) & = & -\overline{u'u'}\,\frac{\partial \overline{u}}{\partial x} & + & \text{DIFFUSION} & + & \text{DISSIPATION} \\ -18 & & -2 & & -13 & & -3 \end{array}$$

The term on the left side of this equation and the first term on the right side (the turbulence production) can be evaluated directly from the data. The dissipation term can be very crudely estimated as 1/3 of the value given by the usual formula for isotropic turbulence, using as dissipation scale the microscale already mentioned. The diffusion term is then the difference. The numbers written above and below the equation are the relative values obtained by this method at Laufer's first station $\left(\frac{x}{d} = 10\right)$ and last

1. The integral scale is defined here as the base of a rectangle having the same area and the same intercept at zero frequency as the measured spectrum in Fig. 16. Laufer's integral scale L is four times smaller because of his conversion to a one-dimensional spectrum by means of formulas derived for isotropic turbulence.

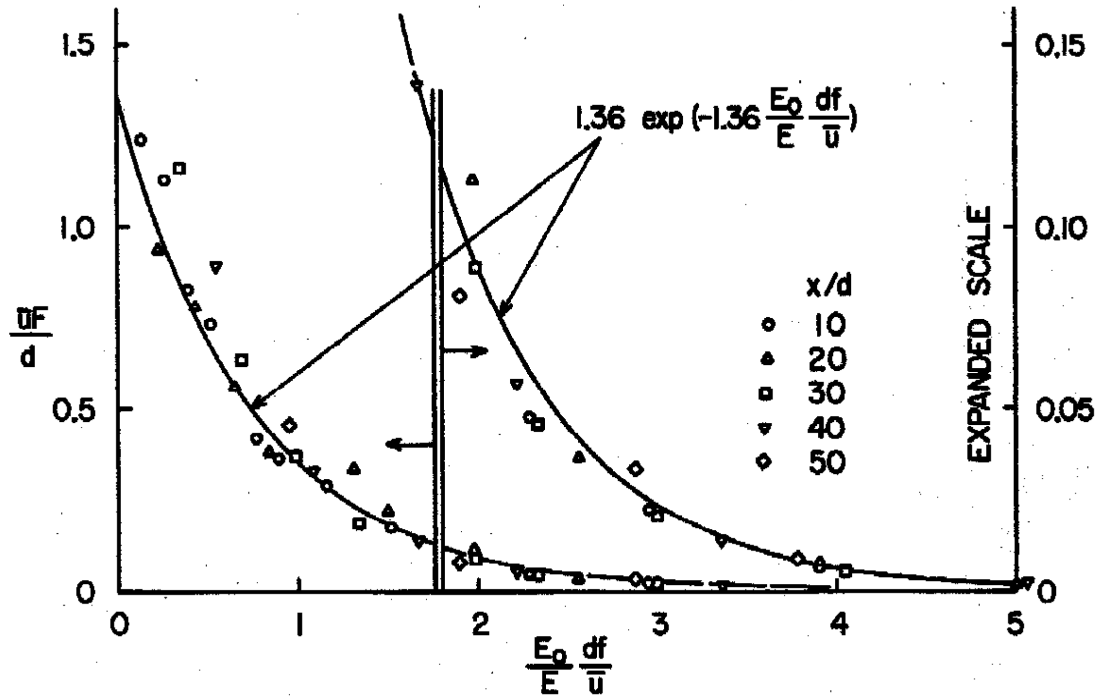


FIGURE 16
Normalized power spectral densities for axial component of decaying shear turbulence in pipe flow at R = 1380 according to Laufer (unpublished).

station $\left(\frac{x}{d} = 50\right)$ respectively. In the early stages of decay, as pointed out by LAUFER, the loss of energy by dissipation is large and is almost balanced by diffusion of energy toward the centerline, much as in uniform pipe flow. In the final stages of decay both terms are greatly diminished, and the diffusion term has changed sign to indicate a flow of energy away from the centerline. This last conclusion is somewhat doubtful, because the length taken as the dissipation scale is of the same order as the pipe diameter. It is not obvious that fluctuations on this scale should be referred to as turbulence in any ordinary sense, or that the flow in question can be usefully discussed in terms of interfaces and intermittency. A conclusion which is not doubtful, on the other hand, is that the turbulence production for the axial component is negative and of substantial size, at least in the early stages of decay. A similar statement can be made, however, for more normal flows such as the wake, and it is necessary to consider the turbulence production on the centerline for the full energy equation. This production is given in the present instance by $\rho\,(-\overline{u'u'}\,\frac{\partial\bar{u}}{\partial x} - \overline{v'v'}\,\frac{\partial\bar{v}}{\partial r} = \rho\,(-\overline{u'u'} + \overline{v'v'}\,)\,\frac{\partial\bar{u}}{\partial x}$ and will be negative only if $\overline{u'u'} > \overline{v'v'}$ (cf. the situation on the centerline of a wake, where this condition is not satisfied).

Finally, let me return to the question of the extreme stability of the mixed laminar-turbulent flows observed in pipes and between rotating cylinders. In these flows there is evidently a balance between production and consumption of turbulent energy such that if the volume of turbulent fluid increases for any reason, the production of turbulent energy increases less rapidly than the consumption; the turbulence is relatively weakened; and the interface velocities are modified so as to reduce the volume of turbulent fluid. This problem can hardly be discussed intelligently, however, without a more complete knowledge of the structure of the turbulent regions in their particular non-turbulent environment. The experimental opportunities in this field are almost unlimited. For example, the natural regularity of the spiral flow configuration described earlier makes it possible to study the interface problem in a captive environment, and an ambitious experimental program is under way at GALCIT to exploit this opportunity. In the large machine already described, the instantaneous values of the three velocity components will be repeatedly sampled at a fixed point in the rotating spiral pattern, and the resulting ensemble of values will be used to obtain the stochastic mean velocity and the six components of the Reynolds stress. The main objective of this experiment is to study the process of energy delivery from the moving walls to the turbulent part of the motion in a typical spiral flow, probably a flow near the point marked " B " in Fig. 8. The measurements, if successful, should help to clarify the question which underlies this whole discussion of intermittency; is the turbulence production $\Phi_t = \tau$ grad $\bar{q}$ a positive definite quantity, corresponding to the dissipation in laminar flow, or is it not ? Other important objectives of the experiment are to determine the average local direction and velocity of propagation of the laminar-turbulent interfaces, and to observe the probability structure of the velocity fluctuations and their products up to perhaps fourth order. Some of these quantities may also be resolved with respect to frequency.

REFERENCES

Binnie, A.M. and Fowler, J.S. (1947). *Proc. Roy. Soc.* **A192**, 32-44.

Coles, D. (1956). *J. Fluid Mech.*, **1**, 191-226.

Corrsin, S. and Kistler, A.L. (1954). *NACA*, TN 3133 (TR 1244, 1955).

Dhawan, S. and Narasimha, R. (1958). *J. Fluid Mech.*, **3**, 418-436.

Elder, J.W. (1960). *J. Fluid Mech.*, **9**, 235-246.

Emmons, H.W. (1951). *J. Aeron. Sci.*, **18**, 490-498.

Klebanoff, P.S. (1954). *NACA*, TN 3178 (TR 1247, 1955).

Liepmann, H.W. and Laufer, J. (1947). *NACA*, TN 1257.

Lindgren, E.R. (1957). *Arkiv för Fysik*, **12**, 1-169.

Lindgren, E.R. (1959a). *Arkiv för Fysik*, **15**, 97-119.

Lindgren, E.R. (1959b). *Arkiv för Fysik*, **15**, 503-519.

Lindgren, E.R. (1959c). *Arkiv för Fysik*, **16**, 101-112.

Lindgren, E.R. (1960a). *Arkiv för Fysik*, **18**, 449-464.

Lindgren, E.R. (1960b). *Arkiv för Fysik*, **18**, 533-541.

Mitchner, M. (1954). *J. Aeron. Sci.*, **21**, 350-351.

Rotta, J. (1956). *Ing.-Arch.*, **24**, 258-281.

Schubauer, G.B. and Klebanoff, P.S. (1955). *NACA*, TN 3489 (TR 1289, 1956).

Schultz-Grunow, F. and Hein, H. (1956). *Z. für Flugwissenschaften*, **4**, 28-30.

Stanton, T.E. and Pannell, J.R. (1914). *Phil. Trans. Roy. Soc.*, **A214**, 199-224.

Taylor, G.I. (1923). *Phil. Trans. Roy. Soc.*, **A223**, 289-343.

Townsend, A.A. (1949). *Australian J. Res.*, **A2**, 451-468.

DISCUSSION

Sir Geoffrey TAYLOR. — LINDGREN's pipe experiments show that when the Reynolds number $Ud/\nu = R$ is confined to a rather short range near 2,400, turbulence is intermittent and in the lower part of this range the turbulent regions are infrequent and widely separated. By careful control of the entry conditions, several experimenters have shown that non-turbulent flow can be maintained at far higher values of R. In some old experiments, I reported non-turbulent flow up to 32,000 and I described how the disturbance produced by local bending of the pipe could be made which produced lateral velocities of order 1 or 2 percent of the mean without inducing turbulence even at $R = 32{,}000$.

What is thought to be the difference between the non-turbulent flow which exists in long lengths of the pipe in LINDGREN's experiments at the lower values of R in the intermittent range and the non-turbulent flow which exists at all values of R, at least up to 32,000, when great care is taken with the entry conditions ?

It is thought that pipe flow is stable for all infinitesimal disturbances and it may be that appropriate kinds of disturbance will grow when the amplitude exceeds a value which depends on the Reynolds number. If this is true, LINDGREN's experiments might be erplained if the distribution of velocity left behind, when the turbulent spot has passed, is more stable than the parabolic distribution, so that larger disturbances would be required to start up turbulent flow just after a turbulent spot has passed than when the distribution has had time to become nearly parabolic again. Thus the lengths of the non-turbulent regions might be determined by the time taken to build up the velocity distribution to a sensitive state. If that were the explanation of LINDGREN's phenomenon, the narrowness of the sensitive range of R might suggest that a comparatively small increase in R would greatly decrease the amplitude of disturbances which would increase, yet the very large increase in R under which permanent non-turbulent flow can exist, even when certain types of disturbances are applied, presents a difficulty to this explanation.

Dr. M. V. MORKOVIN. — The evidence of Dr. LAUFER's and other experiments is that turbulence away from walls dies out very slowly. But in "retransition" observed in turbulent spots on flat plates and in the Couette flow of Dr. COLES, the decay process appears to be rapid. One model of "retransition" which has the feature of rapid decay perhaps should be mentioned — it is probably due to Clauser.

From the motion of the turbulent spots one would judge that the large eddies move with the spots. At the trailing edge of a spot, we are left essentially with smaller-scale turbulence just beyond the laminar sublayer, in presence of an especially stable concave mean-velocity profile. There is no substantial energy input into these smaller eddies and the "buffeting" of the sublayer has been removed. Thus the remnants of turbulence can decay very rapidly in the presence of the wall, without violating our previous views of dying turbulence.

Mr. P. S. KLEBANOFF. — The averaging process of DHAWAN and NARASIMHA to which Dr. COLES refers to is in principle not completely correct. It neglects the non-linear range of instability where the mean flow is distorted and is no longer of the Blasius type. Consequently there are not only the laminar and fully developed turbulent stages but this distorted flow is also present.

As for the lateral propagation of a turbulent spot, this would appear to involve the stability of the flow surrounding it. This can be inferred from the experimental observations which show that the angle of lateral growth depends to some degree on Reynolds number; and that there is a lag in the lateral growth of a turbulent spot when the Reynolds number is below the critical Reynolds number of stability theory.

Author's reply

In response to the comment by Sir Geoffrey TAYLOR, I think it is significant that LINDGREN's flows were initially fully turbulent as a result of disturbances introduced.

by an orifice at the pipe entrance. The emergence of laminar regions farther downstream, therefore, does not necessarily indicate that the laminar profile itself is stable to disturbances coming from adjacent turbulent regions, but only that certain of the profiles produced by momentary local fluctuations in the turbulent velocity distribution may have this property. This question of relative stability for such cases is one which might be investigated both theoretically and experimentally. In particular, I would like to know more about the dynamics of laminar regions embedded in turbulent flows (or vice versa) when these regions are initially forced to take up an abnormal configuration. Sir Geoffrey's suggestion of increasing sensitivity of the laminar profile with increasing distance from an interface, for example, is in effect a suggestion that the length of a laminar region should tend toward a characteristic value at a given Reynolds number. Another experiment worth considering in this connection is a direct determination of the effect of secondary flow on stability and transition, say by connecting two straight sections of pipe by a curved section of variable radius and arc length.

In response to the comment by Dr. Morkovin, I agree that the presence of a wall can greatly accelerate the decay of small-scale turbulent fluctuations near a laminar-turbulent interface; walls are in fact present in all proven cases of retransition. However, it is still necessary to account for the energy of the large eddies and for the kinematic property that the structure of these eddies is apparently not greatly affected by the general straining action of the mean flow.

D. R. Betchov. — Il n'y a vraiment pas de fluides irrotationnels si l'on tient compte de l'agitation thermique (mouvement brownien, etc.).

Celle-ci maintient une structure de petits tourbillons aléatoires qui peuvent à tout instant être amplifiés par les mouvements macroscopiques du fluide. Il est donc possible que la turbulence apparaisse et disparaisse selon que l'écoulement est localement stable ou instable. En électronique, on connaît des oscillateurs à amortissement variable qui se comportent de manière semblable, et amplifient le bruit thermique.

COMMENTAIRE DE LA SECTION : TURBULENCE LIBRE

Prof. Hans W. LIEPMANN, Président

All existing experimental evidence shows that free turbulent flows consist of regions of nearly homogeneous turbulence and irrotational regions. The dividing boundary is sharp and its location a stochastic function of time with scales one order of magnitude larger than the scales of the homogeneous turbulence.

The existence of this "intermittency" removes any hope for a detailed description of free turbulent exchange by a single "Ansatz" for the transport parameters, but it permits the clearer definition of a number of separate problems for study, the solution of which eventually will lead to a complete picture of free turbulence :

a) The effect of strain on homogeneous turbulence. Both theory and experiment are not yet conclusive and present a very promising field for further work.

b) The structure of turbulent interfaces. Interfaces between turbulent and non-turbulent fluid have been encountered in many flow problems and common features can be detected. The experimental evidence is however still very sketchy and little work has been done in which the interfaces were the primary subject. Theoretically and conceptually the two outstanding problems seem to be : (i) the detailed mechanism for the sharply defined edge of turbulent regions, (ii) the possibility for "anti-transition" of not quite randomized disturbances.

c) The instability of turbulent flows. The large scale motion of the interface often bears a striking similarity to instability modes of laminar flow. The investigation of the stability of turbulent flows appears very definitely worthwhile. In particular, experiments with controllable initial disturbances should be done. Important tools for these studies are provided by the recent development of space-time correlation measuring equipment and of apparatus capable of measuring instantaneous velocity profiles.

d) The heuristic model of a non-Newtonian fluid to describe the behaviour of the fine scale turbulent motion seems useful — at least to the writer — to describe the instabilities and transport properties in free turbulent flows.